The Binomial Distribution

Key Vocab

Distribution - the way in which something is shared out among a group or spread over an area

Binomial - where *two* terms are used as part of the calculation

Combinations - the different ways in which a sequence of numbers or letters can be formed

Notation - a series or system of written symbols used to represent numbers, amounts, or elements in something such as music or mathematics

Discrete data that can be counted and only has limited values

Probability - the extent to which an event is likely to occur, measured by the ratio of the favourable cases to the whole number of cases possible

Key Knowledge

Conditions
If a variable is binomially distributed it must satisfy the following conditions:

- Each trial can only have two outcomes (success(p) or failure(q))
- There must be a fixed number of trials (*n*)
- The probability of success (*p*) must be the same for each trial

Notation
We cannot use the binomial formula without n and p
If a random variable X is binomially distributed we use the notation

$$X \sim B(n,p)$$

where n = *number of trials* p = *probability of success* q =*probability of failure*

The Binomial Formula

$$P(X = r) = {}_nC_r \times p^r \times q^{n-r}$$

Example - I DO

Jasper has 3 coins.

In an experiment, Jasper flips each of the 3 coins and records the total number of heads that he gets.

Jasper believes that each coin is biased so that the number of heads he gets can be modelled by the binomial distribution, B(3, 0.4).

(a) Show that P(0 heads) = 0.216

(1)

(b) Work out the probability that the outcome of the experiment is exactly 1 head.

.....

(2)

Example - WE DO

Fruitees sweets come in different flavours.

There are 8 sweets in a pack of mixed flavours and the flavours for each pack are chosen at random.

The mean number of strawberry flavour Fruitees in a pack of 8 sweets is 2

Ed suggests that the number of strawberry flavour Fruitees in a pack of 8 sweets can be modelled by a binomial distribution.

(a) By considering the conditions that make a binomial distribution a suitable model, explain why Ed's suggestion is appropriate.

.....

.....

.....

.....

(2)

One sweet is selected at random from a pack of Fruitees.

(b) Find the probability that the flavour of this sweet is strawberry.

.....

(1)

YOU DO a

The table gives information about blood donations in the UK.

It shows the blood groups O, A, B and AB and the number of donations for each blood group expressed as a percentage of the total number of all donations.

Blood group	O	A	B	AB
Percentage of all donations	48%	38%	10%	4%

(Source: *blood.co.uk*)

6 people attend a clinic on Monday to donate blood.

(a) (i) Name the probability distribution that can be used to model the number of people from these 6 people who will have blood group AB.

.....

(ii) Write down one condition needed so that this distribution is a suitable model.

.....

.....

(2)

(b) Work out the probability that exactly one of these 6 people will have blood group AB.

Give your answer correct to 3 decimal places.

.....

(3)

On Tuesday n people attend the clinic to donate blood.

The probability that at least one of these n people will have blood group AB is greater than 0.5

(c) What can you conclude, if anything, about the value of n?

You must show your working.

.....

.....

.....

(3)

YOU DO c

A biased coin is twice as likely to land Heads as Tails.

An experiment involves flipping 9 identical biased coins.

X represents the number of Heads obtained in the experiment.

Michael says that the values of X can be modelled by the distribution $\mathrm{B}\left(9, \frac{2}{3}\right)$

(a) Write down the name of this distribution model.

.....

(1)

(b) Using Michael's model, show why the mean of X is 6

(1)

The experiment is carried out 15 times and the number of Heads obtained in each experiment is recorded.

Michael uses his model to find an estimate for the number of experiments in which the number of Heads recorded is exactly 6

(c) Find Michael's estimate.

(d) Write down **two** assumptions which Michael needs to make about the experiment so that the distribution $\mathrm{B}\left(9, \frac{2}{3}\right)$ is an appropriate model to use.

.....

.....

.....

.....

.....

.....

(2)

YOU DO d

Nina suggests that each of the following problems can be answered by using a binomial distribution B(n, p).

Problem 1

A manufacturer claims that, in his boxes of balloons, 30% are red, 35% are blue, 20% are green and 10% are yellow.

Find the probability that when 3 boxes are chosen at random exactly 1 box contains no yellow balloons.

Problem 2

Out of 64 matches that Roger Federer has played at Wimbledon, 6 of the matches were five-set matches.

(Source: *www.wimbledon.com*)

Find the probability that, out of the next 8 matches that Roger Federer plays at Wimbledon, exactly 3 of them are five-set matches.

(a) Assess the suitability of using a binomial distribution model to answer each of the problems.

If a binomial model is suitable, you should consider any assumptions that must be made and give possible values of n and p for the model.
If a binomial model is not suitable, you should explain why.

Problem 1

.....

.....

.....

.....

.....

.....

Problem 2

.....

.....

.....

.....

.....

.....

(6)

The probability that a particular machine has a fault is 7%.

A random sample of 6 of these machines are taken and each machine is checked.

(b) Work out the probability that at least 2 of these machines have the fault.

Give your answer correct to 3 decimal places.

.....

(3)