A Level Statistics

AQA Past Exam Questions

TOPIC: Confidence Intervals and the Central Limit Theorem

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions **on paper**

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• **You may use the** booklet 'Statistical Formulae and Tables'

• There are **11** questions in this question paper. The total mark for this paper is **99**

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2018_1
Li Wei is the chief librarian of a public library. As part of a study into library use, he is interested in the times that users spend in the library.
For his study, Li Wei measures the time, x minutes, spent in the library by each of a random sample of 25 users. He finds that:
$$\sum x = 478.5 \text{ and } \sum(x - \bar{x})2 = 698 \ 91$$
Assume that times spent by users in the library may be modelled by a normal distribution.
(a) Construct a 99% confidence interval for the mean time that users spend in the library.

[5 marks]

(b) Li Wei thinks that the interval is too wide to include in a report that he is writing so he decides to take another sample. He still wants to construct a 99% confidence interval.
His assistant, Ranesh, has assured him that if he calculates his interval from a larger sample, then he will certainly end up with a narrower interval than the one constructed in part (a).
Explain why this is not necessarily the case.

[2 marks]

AQA_JUNE_2015_5
(a) (i) A 90% confidence interval for a population mean, μ, is to be constructed. What is the probability that the interval will not include the value of μ?

[1 mark]

(ii) If 5 such confidence intervals are constructed from separate random samples from the same population, find the probability that at least one of them will not include μ.

[3 marks]

(b) Jurgen can run 100 metres in a mean time of 10.325 seconds. His coach changes his training programme to concentrate on his starting speed. After following the new training programme, a random sample of 6 of Jurgen's 100-metre running times has mean 10.280 seconds and standard deviation s = 0.021 seconds.
(i) Assuming Jurgen's 100-metre times are normally distributed, construct a 90% confidence interval for his new mean time to run 100 metres, giving the limits to three decimal places.

[4 marks]

(ii) Use these confidence limits to decide whether there is significant evidence that the new training programme has been effective. Justify your decision.

[2 marks]

AQA_JAN_2013_2
(a) A large survey of adults was designed to find out about sleeping patterns. It included the question 'How many hours sleep do you typically get each night?' The data for the sleeping times of 1403 males, aged between 20 and 60 years, had a mean of 6.75 hours and a standard deviation of 1.29 hours.
(i) Construct a 99% confidence interval for the mean sleeping time of males aged between 20 and 60 years.

(4 marks)

(ii) The 99% confidence interval for the mean sleeping time of females, also aged between 20 and 60 years, was found to be (6.87, 7.05) hours.
Using this confidence interval together with that constructed in part (a)(i), compare the mean sleeping times of males and females aged between 20 and 60 years.

(2 marks)

AQA_JAN_2012_5

The times, in minutes, of a sample of 11 army recruits to complete an assault course were 57 62
74   49   63   65   61   58   79   66   61

(a) Calculate a 95% confidence interval for the mean time for army recruits to complete the course. Assume that any necessary assumptions are satisfied.

(6 marks)

(b) Members of a local football club are invited to use the assault course. It is intended that a random sample of members will be timed and a 90% confidence interval for the mean time will be calculated.

For each of the statements given below, say whether it is:

A Certainly true
B Almost certainly true
C Almost certainly untrue
D Certainly untrue

For those statements to which you have answered B, C or D, explain the reason for your answer.

Statement 1. There is a probability of 0.9 that the confidence interval will contain the mean time taken by members of the club to complete the assault course.

Statement 2. There is a probability of 0.1 that the confidence interval will not contain the mean time taken by members of the sample.

Statement 3. 90% of the times taken by members of the club will be contained in the confidence interval.

(5 marks)

AQA_JUNE_2013_3

Teleza, a psychologist, conducted a series of experiments to investigate the effects of two potential distractors, flashing lights and loud noises, on the time taken by students to perform a simple task. In an initial experiment, a large number of students performed the task without distraction and the mean time was found to be 92 seconds.

Assume that all the task times in this question have normal distributions.

(a) In the next experiment, a random sample of 8 students was subjected to flashing lights, and the times taken to perform the task were recorded by Teleza. These times, in seconds, were as follows.

85   100   105   92   92   89   112   90

(i) Construct a 95% confidence interval for the mean time taken by students to perform the task when subjected to flashing lights, giving the limits to one decimal place.

(6 marks)

(ii) Use this confidence interval to decide whether there is significant evidence that the mean task time has changed from 92 seconds when students are subjected to flashing lights. Justify your decision.

(2 marks)

AQA_JAN_2007_2

Fiona works for a large manufacturing company and each working day she eats in the company's canteen. The times, in seconds, that Fiona spent between entering the canteen and completing the purchase of her meal on eight days were as follows:

248   317   197   233   255   262   321   249

(a) Calculate a 90% confidence interval for the mean time that Fiona spends between entering the canteen and completing the purchase of her meal.

(7 marks)

(b) State two assumptions that are necessary in order for the calculation in part (a) to be valid.

(2 marks)

AQA_JUNE_2017_4

Ruby is the purchasing manager for a clothing factory. She bought rolls of silk, rolls of linen and rolls of cotton from a particular supplier and wanted to check on the mean length of each of these materials on a roll. The length, in metres, of each material on a roll is known to follow a normal distribution.

(a) Ruby measured the length of silk, X metres, on each of a random sample of 5 rolls from a large delivery. She then obtained the following summarised data:

$$\bar{x} = 198.82$$
$$s = 1.96$$

Construct 90% and 99% confidence intervals for μX, the mean length of silk on a roll.

[5 marks]

(b) Ruby plans to construct 90% and 99% confidence intervals for μL, the mean length of linen on a roll.

(i) State the probability that the 90% confidence interval will not contain μL.

(ii) State the probability that both the 90% and 99% confidence intervals will contain μL.

[2 marks]

(c) Ruby also intends to construct 90% and 99% confidence intervals for μC, the mean length of cotton on a roll. However, in this case, the 90% confidence interval will be constructed from one random sample and the 99% confidence interval will be constructed from a second independent random sample.

Find the probability that neither the 90% nor the 99% confidence intervals will contain μC.

[2 marks]

AQA_JAN_2010_5

The lengths, in millimetres, of a random sample of components produced by a particular machine are

135.1 136.9 134.2 133.2 133.7 134.1 133.8 134.7 134.6 134.8

(a) (i) Given that the sample comes from a normal distribution, show that a 95% confidence interval for the mean length of components produced by this machine has limits of 133.78 and 135.24, correct to five significant figures.

(6 marks)

(ii) Explain why more than three significant figures are required for the limits of this confidence interval.

(1 mark)

(b) Read the following three statements, which refer to the population of lengths of components produced by the machine. Ignore the rounding error in the confidence interval quoted in part (a)(i). Indicate, for each statement, whether it is:

A definitely true;
B almost certainly true;
C almost certainly untrue;
or D definitely untrue.

Justify each answer.

Statement 1 Exactly 95% of the population lies between 133.78 and 135.24.

Statement 2 A random sample of size 10 is to be taken from the population. The probability that the mean of this sample will lie in the interval 133.78 to 135.24 is 0.95.

Statement 3 A random sample is to be taken from the population and a 95% confidence interval for the population mean calculated. There is a probability of 0.95 that this confidence interval will contain the mean of this sample.

(6 marks)

AQA_JAN_2011_2

A species of rodent is found in mainland locations and on a remote island.

(a) The weights, x grams, of a random sample of 10 adult males of this species found on the island are recorded as follows.

$$20.4 \quad 23.4 \quad 22.7 \quad 25.9 \quad 20.3 \quad 25.0 \quad 19.7 \quad 23.2 \quad 22.4 \quad 21.5$$

Construct a 95% confidence interval for the mean weight of adult males of this species of rodent in the island location. Assume that weights are normally distributed.

(6 marks)

(b) The weights, y grams, of a random sample of 55 adult males of this species found on the mainland are summarised as follows.

$$\bar{y} = 18.27 \qquad s = 1.638$$

Construct a 95% confidence interval for the mean weight of adult males of this species of rodent in mainland locations.

(3 marks)

(c) Olivia, a zoology student, states that animals living on remote islands are always heavier, on average, than those of the same species living in mainland locations.
Use your confidence intervals to comment on her statement.

(3 marks)

AQA_JUNE_2007_1

In order to economise on office space, a firm decides to encourage its employees to work from home. It claims that employees working from home will save themselves an average of £1250 a year in travelling and other expenses.

A detailed study of 9 employees suggested that the annual amounts, in £, that they would save by working from home were

$$960 \quad 320 \quad 1480 \quad 690 \quad 800 \quad 1650 \quad 1800 \quad 1070 \quad 440$$

(a) Calculate a 95% confidence interval for the mean amount saved by working from home. Assume that the data may be regarded as a random sample from a normal distribution.

(6 marks)

(b) Comment on the firm's claim.

(2 marks)

AQA_JUNE_2010_1

The number of births announced in The Dubworth Weekly Gleaner may be modelled by a Poisson distribution. Last week, 24 births were announced.

(a) Calculate an approximate 99% confidence interval for the weekly mean number of births announced in The Dubworth Weekly Gleaner.

(4 marks)

(b) When the current editor of The Dubworth Weekly Gleaner was appointed ten years ago, there was a weekly mean of 17 births announced. Comment on the editor's claim that the weekly mean number of births announced has increased since his appointment.

(2 marks)