A Level Statistics

AQA Past Exam Questions

TOPIC: The Poisson Distribution

For the new specification, students can now use the Casio Claswiz calculator to find Poisson probabilities. For old AQA questions this was not the case and more work was involved for the students. Therefore, some of the questions will be worth a lot more marks than will be on offer in an up to date Edexcel exam

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **14** questions in this question paper. The total mark for this paper is ***124***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2015_7

Every Saturday evening, Angus runs a disco at the village hall. The hall must be tidied and cleaned on the morning of the following day, Sunday. This is done by Angus and a variable number of volunteers.

Angus keeps a record of the number of volunteers, X, and the probability distribution for X is given in the table.

x	0	1	2	3	4	5	6 or more
P(X = x)	p	0.15	0.20	0.21	0.18	0.14	0

(a) (i) Find the value of p.

(ii) Interpret the implication for Angus of this value of p.

[2 marks]

(b) Find the mean value of X and show that, correct to three significant figures, the standard deviation of X is 1.57 .

[5 marks]

(c) It is suggested that a Poisson distribution may provide an adequate model for X.

(i) Comment on whether your answers in part (b) support this suggestion.

(ii) Give a reason why, in this context, a Poisson distribution may not be an appropriate model.

[3 marks]

AQA_JAN_2012_1

A garage sells batteries, exhausts and tyres.

The number of customers, X, arriving at the garage during a one-hour period to buy a battery may be modelled by a Poisson distribution with mean 2.8 .

The number of customers, Y, arriving at the garage during a one-hour period to buy an exhaust may be modelled by a Poisson distribution with mean 3.5 .

The number of customers, W, arriving at the garage during a one-hour period to buy tyres may be modelled by a Poisson distribution with mean 5.7 .

(a) Find the probability that during a particular hour there are fewer than 2 customers arriving to buy a battery.

(2 marks)

(b) Find the probability that during a particular four-hour period there are exactly 8 customers arriving to buy an exhaust.

(3 marks)

(c) Find the probability that during a particular hour there is a total of more than 15 customers arriving to buy a battery, an exhaust or tyres. Assume that X, Y and W are independent and that each customer buys only one product.

(3 marks)

(d) Explain why the Poisson distribution is unlikely to provide a suitable model for the number of tyres sold during an hour.

(2 marks)

AQA_JAN_2013_5

Alex studies five different subjects at school each weekday, Monday to Friday. The number of pieces of homework, X, which Alex is given each day follows the distribution shown in the table.

x	0	1	2	3	4	5
P(X = x)	0.00	0.03	0.12	0.34	0.33	0.18

(a) Show that the mean of X is 3.51, and calculate the variance of X.

(4 marks)

(b) Find the probability that, on a particular day, Alex is given:

(i) more than 3 pieces of homework;

(1 mark)

(ii) at least the modal number of pieces of homework;

(2 marks)

(iii) fewer than the median number of pieces of homework.

(2 marks)

(c) (i) David, a friend of Alex, suggested that the data in the table could be modelled by a Poisson distribution. Give one reason, apart from a comparison of the values of the mean and the variance found in part (a), why a Poisson distribution would not be a suitable model in this context.

(1 mark)

AQA_JUNE_2013_1

Chris works as the receptionist at the Zubar Dental Practice. The number of requests from male patients for emergency dental treatment that she receives during any one day may be modelled by a Poisson distribution with mean 2.6 .

(a) Find the probability that the number of requests from male patients for emergency dental treatment that Chris receives on a particular day is:

(i) exactly 4;

(2 marks)

(ii) at least 1.

(2 marks)

(b) Find the probability that the number of requests from male patients for emergency dental treatment that Chris receives during a period of 5 days is 15 or fewer.

(2 marks)

(c) The number of requests from female patients for emergency dental treatment that Chris receives during any one day is independent of the number of such requests from male patients and may be modelled by a Poisson distribution with mean 3.4 . The Zubar Dental Practice can provide emergency treatment to a total of 8 patients each day.

Find the probability that Chris receives more requests for emergency dental treatment in one day than the Zubar Dental Practice can provide on that day.

(3 marks)

AQA_JUNE_2014_2

A specimen of radioactive rock emits a-particles, b-particles and g-rays. The a-particles are detected at an average rate of 3.4 per second, the b-particles at an average rate of 2.6 per second and the g-rays at an average rate of 4.0 per second.

Assume that the numbers of a-particles, b-particles and g-rays detected may be modelled by independent Poisson distributions.

(a) Find the probability that:

(i) during a particular second, fewer than 4 a-particles are detected;

[1 mark]

(ii) during a particular period of 5 seconds, more than 10 b-particles but fewer than 20 b-particles are detected.

[4 marks]

(b) Find the probability that:

(i) during a particular second, at least one g-ray is detected;

[2 marks]

(ii) during a particular period of 3 seconds, g-rays will be detected in exactly 2 of the 3 seconds.

[3 marks]

(c) The total number of a-particles, b-particles and g-rays detected during a period of 10 seconds is modelled by a random variable, X. State values for the mean and the standard deviation of X.

[2 marks]

(d) It is discovered that one of the constituents of the rock can emit a b-particle followed within a second by an a-particle. Explain why this would make your answer to part (c) invalid.

[1 mark]

AQA_JUNE_2017_2

A hotel has only two types of ground-floor rooms. There are 4 ground-floor double rooms and 6 ground-floor twin rooms.

The number of requests in one day for ground-floor double rooms, D, may be modelled by a Poisson distribution with a mean of 3.2

The number of requests in one day for ground-floor twin rooms, T, may be modelled by a Poisson distribution with a mean of 3.8

You may assume that the number of requests for ground-floor double rooms is independent of the number of requests for ground-floor twin rooms.

(a) Find the probability that, on one particular day, the hotel:

(i) receives exactly 3 requests for a ground-floor double room;

[2 marks]

(ii) receives at least 2 but fewer than 5 requests for a ground-floor twin room;

[3 marks]

(iii) can satisfy all the requests received for ground-floor double rooms and also for ground-floor twin rooms.

[3 marks]

(b) The hotel aims to be able to offer, on any one particular day, a ground-floor room of some type to every person requesting one, even if it is not of the type requested. How many extra ground-floor rooms must the hotel make available if it is to achieve this aim on at least 99 per cent of days? Justify your answer with probabilities.

[4 marks]

AQA_JAN_2007_1

The number of letters received by Vasha on a weekday may be modelled by a Poisson distribution with mean 5.0 .

(a) Find the probability that on a particular weekday Vasha receives:

(i) 4 or fewer letters;

(ii) exactly 4 letters;

(iii) 4 or more letters.

(5 marks)

(b) Find the probability that the total number of letters received by Vasha over a period of three weekdays exceeds 12 .

(3 marks)

AQA_JAN_2010_5

Guaxara delivers post.

(a) The daily number of items, W, to be delivered to the first house on her round may be modelled by a Poisson distribution with mean 1.4 .

Find the probability that on a particular day she has to deliver 2 or fewer items to the first house on her round.

(1 mark)

(b) The daily number of items, X, to be delivered to the second house on her round may be modelled by a Poisson distribution with mean 3.6 .

Find the probability that on a particular day:

(i) no items are to be delivered to the second house on her round;

(1 mark)

(ii) the number of items to be delivered to the second house on her round is exactly 2;

(2 marks)

(iii) the total number of items to be delivered to the first two houses on her round is more than 6.

(3 marks)

AQA_JAN_2011_2

Mohammed is offered a week's trial with a view to being permanently employed to service bicycles in Robyn's bicycle shop.

The number of bicycles brought in to be serviced may be modelled by a Poisson distribution with mean 2.6 per day.

(a) Find the probability that, on Mohammed's first day, the number of bicycles brought in to be serviced is:

(i) 2 or fewer;

(1 mark)

(ii) more than 3;

(2 marks)

(iii) exactly 4.

(2 marks)

(b) Before starting work, Mohammed told his mother that he hoped that, during his first week (5 days), the number of bicycles brought in to be serviced would be:

* at least 10, otherwise Robyn might decide that there was not enough work to justify permanently employing him;

* not more than 15, so that he would not have to work too hard. Find the probability that Mohammed's hopes will be met.

(4 marks)

AQA_JUNE_2007_1

The number of people entering a supermarket may be modelled by a Poisson distribution with mean 2.4 per minute.

(a) Find the probability that, during a particular minute:

(i) 3 or fewer people enter the supermarket;

(ii) exactly 3 people enter the supermarket.

(4 marks)

(b) Find the probability that, during a five-minute interval, more than 10 people enter the supermarket.

(3 marks)

(c) To pay for their goods, customers must join a queue at one of three checkouts. State, giving a reason, whether it is likely that the number of people per minute joining the queue at a particular checkout may be modelled by a Poisson distribution.

(2 marks)

AQA_JUNE_2008_4

On a map, the symbol P indicates a car park.

A geography student divided a map into 66 squares, each representing an area of 9 km^2 .

(a) If the number of P symbols in a square could be modelled by a Poisson distribution with mean 0.6 , find the probability of a square containing:

(i) no P symbols;

(ii) 3 or more P symbols.

(3 marks)

AQA_JUNE_2009_1

A charity shop opens for 8 hours each day. The daily number of customers requesting a refund for an earlier purchase may be modelled by a Poisson distribution with mean 0.6 .

(a) Find the probability that on a particular day there are:

(i) no requests for refunds;

(ii) more than 2 requests for refunds.

(3 marks)

(b) On a particular day, the shop manager is absent for 4 hours and leaves an assistant in charge of the shop. Find the probability that, during the manager's 4-hour absence, there are:

(i) one or more requests for refunds;

(ii) exactly 2 requests for refunds.

(5 marks)

AQA_JUNE_2010_6

During the football season, an amateur football club holds training sessions for its first team squad on Tuesdays and Thursdays. The number of squad members who do not attend the Tuesday training session may be modelled by a Poisson distribution with mean 3.2 .

(a) Find the probability that, for a particular Tuesday training session:

(i) 6 or more squad members do not attend;

(2 marks)

(ii) the entire first team squad does attend.

(2 marks)

(b) The number of squad members who do not attend the Thursday training session may be modelled by a Poisson distribution with mean 3.8 .

(i) Find the probability that, for a particular Thursday training session, the number of squad members who do not attend is 2 or fewer.

(1 mark)

(ii) Find the probability that, in a particular week, the number of squad members who do not attend the Tuesday training session plus the number of squad members who do not attend the Thursday training session is less than 2. Assume that the number who do not attend on Thursday is independent of the number who do not attend on Tuesday.

(3 marks)

(iii) In an attempt to improve attendance, the club decided to introduce a rule that any squad member who does not attend both training sessions in any week will not be selected for the match on the following Saturday. In the first week that this rule was introduced, the entire first team squad attended on Tuesday and only one squad member did not attend on Thursday.

Do you think the new rule was effective? Explain your answer.

(3 marks)

AQA_JUNE_2011_4

The number of reports of lost credit cards made to a bank's head office may be assumed to follow a Poisson distribution with mean 1.2 per hour.

Find the probability that during a particular hour there will be:

(i) no reports;

(ii) exactly 1 report;

(iii) 5 or more reports.

(5 marks)