A Level Statistics

AQA Past Exam Questions

TOPIC: The Exponential Distribution

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **10** questions in this question paper. The total mark for this paper is ***98***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2012_3

Imran has recently retired and rarely wears a suit. He owns a dark suit which he wears for formal occasions such as weddings. The time, in days, before he next wears the suit may be modelled by an exponential distribution with parameter 0.0045 .

(a) Find:

(i) the mean of this exponential distribution;

(2 marks)

(ii) the probability that Imran will wear the suit during the next 100 days;

(3 marks)

(iii) the probability that Imran will not wear the suit for at least a year (365 days).

(2 marks)

(b) The number of occasions per year on which Imran wears the suit will follow a Poisson distribution. Find the mean of this distribution.

(2 marks)

(c) Imran also owns a light-coloured suit which he wears for social occasions. The number of occasions per year on which he wears the light-coloured suit may be modelled by a Poisson distribution with mean 1.72 .

State the distribution of the number of occasions per year on which Imran wears either his dark suit or his light-coloured suit.

(2 marks)

AQA_JUNE_2013_5

Margaret, the manager of a health centre, is investigating the duration of consultations between patients and doctors.

(a) She initially models the duration, in minutes, of a consultation as a rectangular distribution on the interval [0, 8] .

(i) Using this model, calculate the probability that the duration of a consultation will be between 1 minute and 7 minutes.

(2 marks)

(ii) Find values for the mean and the standard deviation of the duration of a consultation.

(3 marks)

(iii) Give one reason why the above model may not be appropriate.

(1 mark)

(b) Margaret then decides to model the duration, in minutes, of a consultation as an exponential distribution with a parameter of 0.25 .

Using this model, calculate the probability that the duration of a consultation will be:

(i) between 1 minute and 7 minutes;

(3 marks)

(ii) exactly 8 minutes;

(1 mark)

(iii) more than 8 minutes;

(2 marks)

(iv) more than 10 minutes, given that it has already been 8 minutes.

(3 marks)

(c) At the health centre, the appointments given to patients to see a doctor are made at 10-minute intervals. Margaret wishes to reduce this interval to 8 minutes to allow more patients to be seen. The health centre's doctors feel that the current 10-minute time interval is not sufficient and do not want any reduction in the allocated time for a consultation.

Making reference to your answers in part (b), comment on the wishes of either Margaret or the doctors.

(3 marks)

AQA_JUNE_2016_5

The time, T seconds, between the arrival of successive vehicles at a zebra crossing on a road through a village can be modelled by an exponential distribution with parameter $\lambda = 0.025$

(a) Write down the mean and the variance of T.

[2 marks]

(b) Geoffrey is an elderly pedestrian who takes 30 seconds to cross the road using this zebra crossing.

(i) Calculate the probability that no vehicle arrives whilst Geoffrey is crossing.

(ii) Calculate the probability that no vehicle arrives whilst Geoffrey makes two independent crossings.

[3 marks]

AQA_JUNE_2018_2

Julio is a biologist studying a stable population of platypuses that live in burrows in the banks of a long river in eastern Australia.

Julio is modelling the distance in kilometres between platypus burrows with an exponential distribution with parameter $\lambda = 0.36$.

(a) Using this model, find:

(i) the mean distance, in kilometres, between platypus burrows along this river;

[1 mark]

(ii) the probability that Julio has to explore less than 3 km of this river before finding a platypus burrow.

[2 marks]

(b) Julio has already explored 5 km of this river without finding a platypus burrow.

What is the probability that he will have to explore at least a further 2 km before finding a platypus burrow?

[2 marks]

(c) Give one reason, in context, why the exponential model might not be a suitable model for the distance between platypus burrows along the river.

[2 marks]

AQA_JUNE_2017_3

The lifetime of a Waybriter light bulb, used in indoor security lights, is T thousand hours, where T can be modelled by an exponential distribution with parameter $\lambda = 0.0125$

(a) Find the mean lifetime, in hours, of a Waybriter light bulb.

[2 marks]

(b) Find the probability that the lifetime of a Waybriter light bulb is:

(i) less than 100 000 hours;

(ii) between 50 000 hours and 150 000 hours.

[5 marks]

(c) The 15 security lights in a new office block are each fitted with a new Waybriter light bulb. These security lights are then permanently switched on.

Find the probability that all of these 15 light bulbs are working 365 days later.

[3 marks]

AQA_JUNE_2007_4

Adrian is a skilful badminton player. When he serves low, the height, in centimetres, at which the shuttle crosses over the top of the net may be modelled by an exponential distribution with parameter $\lambda = 0.4$

(a) For one of Adrian's low serves, find the probability that the height at which the shuttle crosses over the top of the net is:

(i) less than 2 cm;

(2 marks)

(ii) between 2 cm and 5 cm.

(2 marks)

(b) Verify that, for Adrian's low serves, the median height at which the shuttle crosses over the top of the net is between 1.7 cm and 1.8 cm.

(4 marks)

AQA_JUNE_2008_4

The time to failure, in hours, of a drill bit used in tunnelling machinery may be modelled by an exponential distribution with parameter $\lambda = 0.02$. Drill bits are used continuously until they fail.

(a) Find the mean time to failure of a drill bit.

(2 marks)

(b) Find the probability that a drill bit will fail during an eight-hour shift.

(3 marks)

(c) Find the probability of a drill bit not failing during 5 consecutive eight-hour shifts.

(3 marks)

(d) It is suggested that the risk of a drill bit failing during a shift could be reduced by always using a new one at the start of each shift. Comment on this suggestion.

(2 marks)

(e) The number of drill bits which fail during 5 consecutive eight-hour shifts may be modelled by a Poisson distribution.

(i) Find the mean of this Poisson distribution.

(2 marks)

(ii) Hence find the probability of no drill bits failing during 5 consecutive eight-hour shifts.

(1 mark)

AQA_JUNE_2009_1

Sadia arranges to meet Arlene at a coffee bar on Saturday evenings at 8.00 pm. Past experience suggests that Arlene will arrive at the coffee bar X minutes after 8.00 pm, where X may be modelled by an exponential distribution with parameter 0.05 .

(a) Find the mean and standard deviation of the number of minutes after 8.00 pm that Arlene will arrive at the coffee bar.

(2 marks)

(b) If Sadia arrives at 8.20 pm, find the probability that Arlene will already have arrived.

(3 marks)

(c) Sadia arrives at 8.20 pm and finds that Arlene has not yet arrived. Find the probability that Arlene will arrive after 8.30 pm.

(3 marks)

AQA_JUNE_2010_1

Rowena's sewing machine occasionally misses a stitch. When Rowena sews a seam, the distance, X metres, to the first missed stitch may be modelled by an exponential distribution with mean 0.8 .

(a) Show that the parameter, λ , of this distribution is 1.25 .

(1 mark)

(b) Find the probability that the machine misses a stitch before Rowena has sewn 0.5 metres of a seam.

(3 marks)

(c) Rowena has to sew a seam of length 1.4 metres.

(i) Find the probability that no stitches have been missed when she has sewn half of this seam.

(3 marks)

(ii) Assuming that no stitches were missed in the first half of the seam, find the probability that there is at least one missed stitch in the completed seam.

(2 marks)

AQA_JUNE_2011_4

The time, in minutes, that Michaela waits at a bus stop to catch a bus to work may be modelled by an exponential distribution with parameter $\lambda = 0.22$.

(a) Find the mean and the standard deviation of the time that Michaela waits for a bus.

(2 marks)

(b) Find the probability that Michaela will have to wait more than 5 minutes for a bus.

(3 marks)

(c) On Monday morning, Michaela, who has already been waiting at the bus stop for 3 minutes, is joined by a friend, Narinder. Find the probability that Narinder will have to wait less than 3 minutes for a bus.

(3 marks)