A Level Statistics

AQA Past Exam Questions

TOPIC: The Exponential Distribution

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **10** questions in this question paper. The total mark for this paper is ***98***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2012_3

3(a)(i)	mean = 1/0.0045 = 222.2	M1 A1	 2	M1 method A1 222 (222~222.4)
(ii)	probability will wear the suit in next 100 days $= 1 - e^{-0.45}$ $= 1 - 0.638 = 0.362$	 M1 m1 A1	 3	 M1 100×0.0045 m1 method - allow wrong tail A1 0.362 (0.362~0.363)
(iii)	probability will not wear suit for a year = $e^{-365\times0.0045}$ $= e^{-1.6425}$ = 0.193	 M1 A1	 2	 M1 method - allow wrong tail A1 0.193 (0.193~0.194)
(b)	mean = 365 × 0.0045 = 1.64	M1 A1	 2	M1 method A1 1.64 (1.64~1.65)
(c)	number of times per year which Imran wears a suit is Poisson mean 1.64 + 1.72 = 3.36	 B1 B1	 2	 B1 Poisson, mean 1.72 + their (b) B1 3.36 (3.36~3.37)
	Total		**11**	

AQA_JUNE_2013_5

Q	Solution	Marks	Total	Comments
5(a)(i)	$P(1\le X\le 7)=\frac{7-1}{8}=0.75$	M1 A1	 2	M1: using correct rectangular distribution, allow slip eg 7/8 or 5/8
(ii)	Mean = 4 mins	B1		
	Standard deviation = $\sqrt{\frac{(8-0)^2}{12}}=2.31$	M1A1	3	A1 awfw 2.30 ~ 2.31 s.c B1 for $\frac{64}{12}$
(iii)	Under this model it is impossible for a consultation to last longer than 8mins	B1	1	
(b)(i)	F(7) – F(1) =	M1		M1: sight of $1-e^{-\frac{7}{4}}$ or $1-e^{-\frac{1}{4}}$ or 1 – 0.1738 = 0.8262 or 1 – 0.7788 = 0.2212
	$\left(1-e^{\frac{-7}{4}}\right)-\left(1-e^{\frac{-1}{4}}\right)$	m1		m1: subtracting their F(7) – their F(1)
	= 0.605	A1	3	awfw 0.60 ~ 0.61
(ii)	$P(X=8)=0$	B1	1	
(iii)	$P(X\ge 8)=1-F(8)=1-\left(1-e^{\frac{-8}{4}}\right)$	M1		
	= 0.135	A1	2	awfw 0.135 ~ 0.136
(iv)	$P(X\ge 10 / X\ge 8)=P(X\ge 2)$	M1		Using "no memory"property
	= 1 – F(2)	M1		
	= 0.61	A1	3	A1 awfw 0.60 ~ 0.61 accept $e^{-0.5}$
	or $P(X\ge 10 / X\ge 8)=\frac{P(X\ge 10)}{P(X\ge 8)}$			
	$=\frac{e^{\left(\frac{-10}{4}\right)}}{e^{\left(\frac{-8}{4}\right)}}$	(M1)		M1 Numerator and dividing
		(M1)		M1 denominator
	= 0.61	(A1)	(3)	A1 awrt 0.60 ~ 0.61 accept $e^{-0.5}$
				NB: must use correct probability distribution in all parts above

Q	Solution	Marks	Total	Comments
5(c)	Under new system 13.5% of appointments would overrun and of these approx 61% would take longer than 10 minutes. Reduction in appointment time is likely to make patients wait – doctors' wishes are supported. Note: the use of expressions such as "likely" or "most" must be supported by a numerical probability. scE1 for answers unsupported by correct numerical evidence	B2 E1	 3	B1 for each distinct correct numerical comment on probabilities using the exponential model to a maximum of 2. E1 A single conclusion supported by numerical comments dependent on at least one B1. Alternatives: Approx. 60% of consultations last between 1 and 7 minutes and only 13.5% take longer than 8 minutes. Health centre's suggestion is reasonable; Margaret's wishes are supported. or unlikely almost 22% of all appointments last less than 1 minute poor model – more research needed
	Total		**18**	
	TOTAL		**75**	

AQA_JUNE_2016_5

5a	Mean $= \frac{1}{\lambda} = 40$; variance $= \left(\frac{1}{\lambda}\right)^2 = 1600$	B1,B1	**2**	Cao both
5b(i)	$P(T > 30) = e^{-0.025\times30}$ $= 0.4724$	M1 A1		or 1 - $(1- e^{-0.025\times30}) = 1 - 0.528$ awfw 0.472 ~ 0.473 (0.472366…)
b(ii)	On 2 occasions : prob $= 0.4724^2$ $= 0.2231$	B1ft	**3**	awrt 0.223 ~ 0.224: f.t. on their b (i)
(c)	$P(\bar{T} > 35) = P(Z > \frac{35-40}{\sqrt{\frac{1600}{75}}})$	M1		Standardising with 35 and 40 ; condone $\sqrt{40}$ or $\frac{1600}{75}$ as denominator.
		B1		$\sigma = \sqrt{\frac{1600}{75}}$ *or* $\sigma^2 = \frac{1600}{75}$ seen or implied by correct probability.
	$= P(Z > -1.08..)$	A1		[awrt 4.62 (4.6188..)] awfw –1.08 ~ – 1.09
	$= 0.860$	A1		0.859 ~ 0.863 (0.86049…) NMS 4/4 for a probability in correct range.
			4	
	Total		**9**	

AQA_JUNE_2018_2

(a) (i)	$\mu = \frac{1}{\lambda} = \frac{1}{0.36} = 2.78$	B1	1	Or 2.77
(ii)	$1 - e^{-0.36\times3}$	M1		PI Correct use of formula
	$= 0.660$ or 0.66	A1	2	AWRT (0.6604044744)
(b)	(*Use* of memoryless property) x = 2 $1-(1-e^{-0.36\times2})$ or $e^{-0.36\times2}$	M1		*Not* using both $1-e^{-0.36\times2} = 0.513$ x = 2 and x = 5 Allow correct use of conditional probability with x = 5 and x = 7.
	= 0.487 or 0.486	A1	2	AWRT (0.486752256)
(c)	***Either*** The locations of platypus burrows might not be independent (of each other).	B1 E1	2	PI not independent or equivalent random clear and correct context (Might be stated as "platypus burrows may be grouped together" or "platypus burrows might be at a fixed distance apart".) "platypuses live in colonies"
	or Platypus burrows might not occur at a constant average rate/interval along the river	(B1) (E1)		not constant average rate/interval clear and correct context For E1 must have mentioned burrows/homes not just platypuses
	Total		**7**	

AQA_JUNE_2017_3

3(a)	Mean $= \frac{1}{\lambda} = \frac{1}{0.0125}$ $= 80$ Mean lifetime is 80 000 hours	 M1 A1	**2**	$\frac{1}{0.0125}$ cao s.c. B1 for 80 with no other working shown.
3(b)(i)	$P(T < 100) = \left(1 - e^{-\frac{100}{80}}\right)$ $= (1 - e^{-1.25})$ $= 0.713$	B1 M1 A1		Using T = 100 Use of $F(t) = 1 - e^{-0.0125t}$ with their T 0.71~ 0.72 (0.713495…)
(ii)	$P(50 < T < 150) = P(T < 150) - P(T < 50)$ $= \left(1 - e^{-\frac{150}{80}}\right) - \left(1 - e^{-\frac{50}{80}}\right)$ $= e^{-0.625} - e^{-1.875}$ $= 0.53526 - 0.15335$ $= 0.382$	 M1 A1	**5**	Subtracting two valid cumulative probabilities o.e.; ft their λ ; must be using T =150 and T = 50; allow small slip. or 0.84665 – 0.46474 0.38 ~ 0.39 (0.38190…)
3(c)	365 days = 365 × 24 hours = 8760 P(all bulbs last longer than 8760 hours) $= \left(e^{-\frac{8.76}{80}}\right)^{15} = (0.89628)^{15}$ o.e. eg $e^{-0.0125\times8.76\times15}$ $= 0.193$ Alt: 365×24×15=131400 **M1**; P(T>131.4) = $e^{-0.0125\times131.4}$ **m1** = 0.193 **A1**	 M1,m1 A1	**3**	M1;dividing their calculated hours by 1000 and using $e^{-0.0125t}$ m1 raising their probability to the power of 15. 0.193 ~ 0.194 (0.1934886…)
	Total		**10**	

AQA_JUNE_2007_4

4(a)(i)	$P(X < 2) = 1 - e^{-0.4\times 2}$ $= 1 - e^{-0.8} = 0.551$	M1 A1	2	or by integration AWRT
(ii)	$P(2 \le X \le 5) = F(5) - F(2)$ $= (1 - e^{-2}) - (1 - e^{-0.8})$ $= 0.314$	M1 A1	2	or by integration AWRT
(b)	for median m, $F(m) = 0.5$ $(= 1 - F(m))$ $F(1.7) = 1 - e^{-0.68} = 0.493$ $\left(e^{-0.68} = 0.507\right)$ $F(1.8) = 1 - e^{-0.72} = 0.513$ $\left(e^{-0.72} = 0.487\right)$ 0.5 lies between 0.493 and 0.513 so median lies between 1.7 and 1.8 or $e^{-0.4m} = 0.5$ $-0.4m = \ln(0.5)$ $m = \dfrac{0.693}{0.4} = 1.73$ so median lies between 1.7 and 1.8	B1 B1 B1 E1 (M1) (m1) (A1) (E1)	4	may be implied equation of correct form attempt to solve using logs solution used to answer question
	Total		**8**	

AQA_JUNE_2008_4

4(a)	mean $\dfrac{1}{0.02}$ = 50 hours	M1 A1	2	method 50 CAO - ignore units
(b)	$1 - e^{-8 \times 0.02} = 1 - e^{-0.16}$ $= 1 - 0.8521437$ $= 0.148$	B1 M1 A1	3	attempt to use $e^{-8 \times 0.02}$ correct method 0.148 (0.1475 ~ 0.1485)
(c)	Probability not fail during 40 hours $1 - e^{-0.8} = 1 - 0.4493$ $= 0.551$ Probability not failing = 0.449 (or $0.8521437^5 = 0.449$)	M1 m1 A1	3	attempt to find probability not failing during 40 hours or (their prob not fail in 8 hours)^5. Allow fail/not fail errors correct method 0.449 (0.449 ~ 0.45)
(d)	Makes no difference - exponential distribution has no memory.	E1 E1	2	no difference exponential distribution has no memory
(e)(i)	Mean time between failures is 50 hours. Mean number of drill bits which fail in 40 hours is $\dfrac{40}{50} = 0.8$	M1 A1	2	method 0.8 CAO
(ii)	From tables (or otherwise) 0.449	B1	1	0.449 (0.449 ~ 0.45)
	Total		**13**	

AQA_JUNE_2009_1

1(a)	mean $1/0.05 = 20$ s.d. $1/0.05 = 20$	M1 A1	 2	Method for both 20 both, CAO
(b)	$1 - e^{-0.05 \times 20}$ $= 1 - e^{-1}$ $= 0.632$	B1 M1 A1	 3	0.05×20 Method - allow wrong tail 0.6315 ~ 0.6325
(c)	$e^{-0.05 \times 10}$ $= e^{-0.5}$ $= 0.607$	M1 m1 A1	 3	Attempt to find > or < 10 from exponential parameter 0.05 or equivalent Method - allow wrong tail 0.606 ~ 0.607
	Total		**8**	

AQA_JUNE_2010_1

1(a)	$\lambda =$ 1/mean $= 1/0.8$ $= 1.25$	E1	1	E1 1/0.8 ag
(b)	$P(X < 0.5) = 1 - e^{-1.25 \times 0.5}$ $= 1 - e^{-0.625} = 1 - 0.535$ $= 0.465$	B1 M1A1	 3	B1 1.25×0.5 M1 method – allow wrong tail A1 0.465 (0.464 ~ 0.466)
(c)(i)	$P(X > 0.7) = e^{-1.25 \times 0.7}$ $= e^{-0.875}$ $= 0.417$	M1 m1 A1	 3	M1 attempt to find > 0.7 from exponential parameter 1.25 m1 method – allow wrong tail A1 0.417 (0.416 ~ 0.418)
(ii)	$P(X < 1.4 \mid X > 0.7)$ $= P(X < 0.7)$ $= 1 - 0.417 \quad = 0.583$	 M1 A1	 2	 M1 1 – their (c)(i) A1 0.583 (0.582 ~ 0.584)
	Total		**9**	

AQA_JUNE_2011_4

4(a)	mean $\frac{1}{0.22} = 4.55$ sd $\frac{1}{0.22} = 4.55$	M1 A1	 2	for both 4.55 (4.54 ~ 4.55) for both
(b)	$e^{-0.22 \times 5}$ $= e^{-1.1}$ $= 0.333$	B1 M1 A1	 3	0.22×5 allow wrong tail 0.333 (0.332 ~ 0.333)
(c)	$1 - e^{-0.22 \times 3}$ $= 1 - e^{-0.66}$ $= 1 - 0.5168$ $= 0.483$	M1 m1 A1	 	attempt to find > or < 3 from exponential parameter 0.22 allow wrong tail 0.483 (0.483 ~ 0.484)