A Level Statistics

AQA Past Exam Questions

TOPIC: Discrete Random Variables

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions **on paper**

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• *You may use the* booklet 'Statistical Formulae and Tables'

• There are **17** questions in this question paper. The total mark for this paper is *158*

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2015_7

Every Saturday evening, Angus runs a disco at the village hall. The hall must be tidied and cleaned on the morning of the following day, Sunday. This is done by Angus and a variable number of volunteers.

Angus keeps a record of the number of volunteers, X, and the probability distribution for X is given in the table.

x	0	1	2	3	4	5	6 or more
P(X = x)	p	0.15	0.20	0.21	0.18	0.14	0

  a)

      i.   Find the value of p.

      ii.   Interpret the implication for Angus of this value of p.

[2 marks]

  b) Find the mean value of X and show that, correct to three significant figures, the standard deviation of X is 1.57 .

[5 marks]

AQA_JAN_2012_3

A restaurant offers fresh lobster on its menu. On each lobster sold, the restaurant makes a profit of £24 . On each lobster unsold at the end of the day, it makes a loss of £16 . At present, the manager buys four lobsters each day. Past experience has shown that the number of lobsters, X, requested daily by customers follows the distribution shown in Table 1.

Table 1

x	0	1	2	3	4
P(X = x)	0.1	0.15	0.25	0.35	0.15

  a)

      i.   Show that the mean value of X is 2.3 , and calculate the standard deviation of X.

(4 marks)

      ii.   Use this mean value of X and the corresponding mean value of lobsters unsold to show that the restaurant's mean daily profit on lobsters is £28 .

(2 marks)

  b) The manager considers reducing the number of lobsters that she buys each day to three. Assuming that the distribution of the number of lobsters requested daily by customers stays the same, Table 2 shows the distribution of the number of lobsters, Y, sold daily under this new arrangement.

Table 2

y	0	1	2	3
P(Y = y)	0.1	0.15	0.25	k

      i.   State the value of k.

(1 mark)

      ii.   Calculate the mean daily profit from the lobsters when the manager buys only three each day.

(3 marks)

      iii. Give one advantage of the manager reducing the number of lobsters that she buys to three each day.

(1 mark)

      iv. Give one disadvantage of the manager reducing the number of lobsters that she buys to three each day.

(1 mark)

AQA_JAN_2013_5

Alex studies five different subjects at school each weekday, Monday to Friday. The number of pieces of homework, X, which Alex is given each day follows the distribution shown in the table.

x	0	1	2	3	4	5
P(X = x)	0.00	0.03	0.12	0.34	0.33	0.18

    a) Show that the mean of X is 3.51, and calculate the variance of X.

(4 marks)

    b) Find the probability that, on a particular day, Alex is given:
       i.   more than 3 pieces of homework;

(1 mark)

       ii.   at least the modal number of pieces of homework;

(2 marks)

       iii. fewer than the median number of pieces of homework.

(2 marks)

AQA_JUNE_2012_1

Past experience shows that, when they play home matches, Newville Athletic football team win half the matches and lose 20% of the matches. They draw the remaining home matches. The team is awarded three points for a win, one point for a draw and no points for a loss.

    a) The table below shows the probability distribution of X, the number of points awarded to Newville Athletic after a home match. Complete the table.

(2 marks)

x	0	1	2	3
P(X =x)				0.5

    b) Calculate the mean and the standard deviation of the number of points awarded to Newville Athletic after a home match.

(5 marks)

    c) State the probability that the number of points awarded to Newville Athletic after a home match is more than one standard deviation above the mean.

(1 mark)

AQA_JAN_2008_5

A library has branches in Molcar and Garsden. The Molcar branch has 4 computer terminals available for the use of library members. Muttiah, a librarian, records the number of terminals, X, in use at random times during weekdays. He observes that X may be modelled by the following probability distribution:

X	P(X = x)
0	0.005
1	0.015
2	0.080
3	0.150
4	0.750

    a)
       i.   Show that the mean of X is 3.625 .
       ii.   Find the standard deviation of X.

(5 marks)

AQA_JUNE_2013_3

bank has an ATM (Automated Teller Machine) which customers can use to withdraw cash. Withdrawals can be made in fixed amounts, £ X. The table shows the amounts available and the probability distribution for X.

x	P(X = x)
10	0.18
20	0.44
50	0.13
100	0.08
200	0.17

a)
- i. Find the value of E(X).

  (2 marks)
- ii. Show that the standard deviation of X is 68.0, correct to three significant figures.

  (3 marks)
- iii. Find the probability that at least one out of a random sample of three customers withdraws more than the mean amount of cash.

  (3 marks)

b) The bank is considering making an additional amount of £300 available. It is expected that some of the customers who currently withdraw £200 would then withdraw £300. State whether this change would increase, decrease or leave unchanged:
- i. the mean of X ;
- ii. the standard deviation of X.

  (2 marks)

c) A small number of customers use the ATM to see the balance in their account and do not withdraw any cash. If the table were changed to include these customers, explain why this would decrease the mean of X.

  (2 marks)

AQA_JUNE_2014_1

Todd is a dentist. Clients at Todd's surgery pay one of three possible fees: £20 for a check-up only, £50 for a check-up followed by minor treatment, and £210 for a check-up followed by major treatment. Experience shows that the probabilities for those needing treatment are as in the table.

	Fee	Probability
Check-up only	£20	
Check-up + minor treatment	£50	0.32
Check-up + major treatment	£210	0.11

a)
- i. Write down the probability for clients needing a 'Check-up only'.

  [1 mark]
- ii. Todd wants to draw a pie chart for his surgery wall to illustrate the data in the table. Calculate, to one decimal place, the angles that he should use for the three sectors.

  [2 marks]

b) Show that the mean amount paid by Todd's clients is £50.50 and find the standard deviation of the amount paid.

  [4 marks]

c) At present, Todd sees an average of 90 clients each week. He believes that he will increase this number by 20 per cent if he reduces the charge for a 'Check-up only' to £10, leaving the other fees unchanged. Assuming that this belief is true and that the probabilities remain unaltered, find:
- i. the new mean amount paid by Todd's clients;
- ii. the increase in Todd's average weekly income from his clients.

  [3 marks]

AQA_JUNE_2016_2

A ticket purchased at a car park allows parking for a period of up to 1, 2, 3, 4, 8 or 24 hours. The ticket must be displayed in the parked car. The percentage of purchases of each type of ticket are shown in the table.

Type of ticket	Percentage
Up to 1 hour	12
Up to 2 hours	19
Up to 3 hours	18
Up to 4 hours	21
Up to 8 hours	10
Up to 24 hours	20

a)
  i. Find the probability that the ticket displayed in a randomly chosen car at this car park allows parking for a period of more than 3 hours.

[1 mark]

  ii. There are two yellow cars in the car park each displaying a ticket. Calculate the probability that the two tickets displayed are of the same type.

[2 marks]

b) The charge for parking at this car park is 50p per hour for any period up to 4 hours. There are fixed charges of £3 for up to 8 hours and £5 for up to 24 hours.

  i. Find the mean amount paid for a parking ticket at this car park and show that the standard deviation is £1.53 correct to three significant figures.

[5 marks]

  ii. Find the probability that the amount paid for a parking ticket is within one standard deviation of the mean.

[2 marks]

AQA_JUNE_2018_1

A ferry company has recorded for many years the number of people in each car that it transports. The table shows the percentage of cars for each number of people in a car.

Number of people in car	1	2	3	4	5	6 or more
Percentage of cars	24	36	15	22	3	0

It may be assumed that the number of people in a car is independent of the number of people in any other car.

a) Find the probability that a randomly chosen car transported on the ferry contains more than 2 people.

[1 mark]

b) On a particular sailing there are two green cars on the ferry. Calculate the probability that one green car contains fewer than 3 people and the other green car contains more than 3 people.

[2 marks]

c) Calculate the mean number of people per car transported by the ferry, and show that the standard deviation is 1.16 , correct to three significant figures.

[5 marks]

d) The charge, £C, for transport using the ferry is £68 for a car and driver, and an extra £24 for each additional person in the car. Find the mean and the standard deviation of C.

[2 marks]

AQA_JUNE_2017_4

Petra sells lockets on her market stall. For a fee, she will engrave letters on a locket when she sells it.

The table shows the percentage of Petra's total locket sales for each number of letters engraved on a locket.

Number of letters	0	1	2	3	4	5	6	7	8	>8
Percentage of lockets	26	18	6	9	12	13	9	5	2	0

a)
  i. Find the probability that the next customer who buys a locket from Petra asks her to engrave 4 or more letters on the locket.

  [1 mark]

  ii. One morning, Petra sells 3 lockets. Calculate the probability that she engraves some letters on exactly 2 out of these 3 lockets.

  [3 marks]

  iii. Find the mean number of letters engraved on a locket and show that the variance is 5.81, correct to three significant figures.

  [4 marks]

b) The price of a locket is £5 and Petra charges 40 pence per letter for engraving.
  i. Find the mean cost of the lockets sold by Petra.

  [2 marks]

  ii. Find the standard deviation of the cost of the lockets sold by Petra, giving your answer to the nearest penny.

  [2 marks]

AQA_JAN_2007_3

Habib shops at his local store. The store sells four different brands of instant coffee in 100-gram jars but not all are available at all times.

When Habib buys instant coffee, he chooses the Fairtrade brand if it is available. If it is not available, his preferences, in order, are Own Brand, Brand A, Brand B.

He spends X pence on a 100-gram jar of coffee. The probability distribution shows the price, x pence, of each brand together with the probability that Habib will buy that brand.

Brand	x	P(X = x)
Fairtrade	225	0.56
Own Brand	145	0.32
Brand A	249	0.09
Brand B	253	0.03

a) Show that the mean value of X is 202.4 .

(2 marks)

b) Find the standard deviation of X .

(3 marks)

c) State the mean and standard deviation of the amount Habib would spend on a 100-gram jar of instant coffee if all brands were available at all times.

(2 marks)

d) Give one advantage to the store owner of having all brands available at all times.

(1 mark)

A convenience store stocks three brands of teabags which are sold in boxes of 40.
The price, X pence, of a box and the probability that a customer buying teabags will choose that brand is shown in Table 1.

Table 1

	X	P(X = x)
Own brand	99	0.50
Fairtrade	125	0.30
Supertea	144	0.20

a) Find the mean and the standard deviation of X.

(4 marks)

b) Alec, the store owner, is considering stocking a cheaper brand of teabags, Sweepings, which would sell for 79p per box of 40. He estimates that, if he did this, there would be a 20 per cent increase in the number of boxes of teabags sold.
The new distribution of prices, Y pence, would be as shown in Table 2.
Table 2

y	P(Y = y)
Sweepings 79	0.250
Own brand 99	0.375
Fairtrade 125	0.225
Supertea 144	0.150

   i.   Show that the mean value of Y is 107, correct to the nearest penny.

(2 marks)

   ii.   Assume that Alec's estimates are correct. Determine whether, if Alec stocks Sweepings, the increased sales would, despite the reduced average price, lead to an increase in the total amount of money taken on teabags.

(3 marks)

c) Apart from a possible increase in the total amount of money taken on teabags, suggest one advantage of stocking Sweepings.

(1 mark)

Imran wishes to buy a house in Cheadleville. The number of houses, X, in Cheadleville advertised for sale in a copy of the Cheshire Weekly Sentinel may be modelled by the following probability distribution.

x	0	1	2	3	4	5
P(X=x)	0.32	0.25	0.19	0.12	0.09	0.03

a) Find the mean and the standard deviation of X.

(5 marks)

b) The number of houses in Cheadleville advertised for sale in a copy of the Cheshire Weekly Clarion may be modelled by the random variable Y.
Given that E(Y) = 2.5
and $E[(Y - 2.5)^2] = 2.2$:
   i.   evaluate the standard deviation of Y;

(2 marks)

   ii.   compare the number of houses in Cheadleville advertised for sale in the Cheshire Weekly Sentinel with that in the Cheshire Weekly Clarion.

(2 marks)

c) Imran intends to subscribe to one of the two papers. Advise him which one to choose, justifying your answer.

(2 marks)

AQA_JUNE_2008_2

A county cricket club has different categories of membership. The following table shows the categories of membership, the annual subscription for each category of membership and the probability that a new member will join that category.
Category of membership Annual subscription,

	£	Probability
Full	120	0.22
Senior	80	0.28
Country	75	0.12
Junior	30	0.38

a)
    i. Show that the mean subscription paid by new members is £69.20 .
    ii. Find the standard deviation of the subscription paid by new members.

(5 marks)

b) As the ground capacity is limited, only 400 new members can be accepted during the coming year. The club is to stage an international match and so expects more than 400 applications for membership.
It is decided that only applications for full membership will be accepted.
How many full members would the club need to accept in order to receive more money in subscriptions than would have been provided by 400 members distributed as in the table above?

(2 marks)

c) Give one disadvantage of the decision in part (b).

(1 mark)

AQA_JUNE_2009_2

Nazia owns an ice-cream van and sells tubs containing 1, 2, 3 or 4 scoops of ice cream. The prices, X pence, of these tubs together with the distribution of sales are shown in Table 1.

Table 1

	x	P(X = x)
1 scoop	50	0.40
2 scoops	95	0.16
3 scoops	135	0.24
4 scoops	170	0.20

a) Find the mean and the standard deviation of X.

(4 marks)

b) Nazia is considering discontinuing the sale of tubs containing 1 scoop of ice cream. She estimates that if she does this:
* half of those customers who buy such tubs would instead buy tubs containing 2 scoops of ice cream;
* the other half would not buy any ice cream.
The new distribution of prices, Y pence, would be as shown in Table 2.

Table 2

y	P(Y = y)	
2 scoops	95	0.45
3 scoops	135	0.30
4 scoops	170	0.25

    i. Show that the mean value of Y is 126, correct to three significant figures.

(2 marks)

    ii. By making a suitable calculation, advise Nazia as to whether, if her estimate is correct, she will increase the total amount of money taken if she discontinues selling tubs containing 1 scoop of ice cream.

(3 marks)

AQA_JUNE_2010_1

The table shows the different possible fares, X pence, and the probability that they apply to a fare-paying passenger boarding a bus.

Fare, x pence	P(X = x)
40	0.37
70	0.18
100	0.14
140	0.12
190	0.19

a)
  i.   Find the mean value of X.
  ii. Show that $E(X^2) = 12\,085$.
  iii. Find the standard deviation of X.

(5 marks)

b) Find the probability that a fare-paying passenger boarding the bus pays:
  i.   more than £1.20 ;
  ii. exactly £1.20 .

(2 marks)

c) The table only includes passengers who pay fares. Children under 5 and holders of a senior citizen bus pass travel free. State, giving a reason, whether, if the zero fares of these passengers were included, the mean would be larger, smaller or the same as that calculated in part (a).

(2 marks)

AQA_JUNE_2011_2

In a pay-and-display car park, users are charged different amounts according to the lengths of time they wish to park their cars.
The following table shows the distribution of the amounts, X pence, paid by users.

x	P(X = x)
100	0.22
200	0.31
300	0.21
400	0.12
600	0.14

a)
  i.   Show that the mean of X is 279 .
  ii. Find the standard deviation of X.

(4 marks)

b) A small proportion of the cars in the car park belong to employees of the firm which operates the car park, who are allowed to park their cars with no charge. If these employees were included as users, state, giving a reason, whether the standard deviation would increase, stay the same or decrease.

(2 marks)

c) In fact, there is no charge for cars entering the car park after 6 pm. In the evening, it is used by a large number of people going to a nearby cinema. At 9 pm, a few cars which have been parked before 6 pm remain in the car park, but nearly all the cars in the car park have parked with no charge. State, giving a reason, whether the standard deviation of the amounts paid to park the cars which are in the car park at 9 pm is greater than, the same as or less than the standard deviation of X.

(2 marks)