

A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing

Paired t Test

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions **on paper**
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- **You may use the** booklet 'Statistical Formulae and Tables'
- There are **8** questions in this question paper. The total mark for this paper is **72**
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

Q1	Solution					Marks	Total	Comments																								
(a)	H_0 pop mean diff $\mu_d = 0$ H_1 pop mean diff $\mu_d > 0$ 1 tail 1% $d = \text{Robusta} - \text{Arabica}$					B1		Hypotheses																								
	<table border="1"> <thead> <tr> <th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr> </thead> <tbody> <tr> <td>d</td><td>133</td><td>104</td><td>-87</td><td>-86</td><td>142</td></tr> <tr> <td></td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td></tr> <tr> <td>d</td><td>101</td><td>185</td><td>167</td><td>209</td><td>97</td></tr> </tbody> </table>						A	B	C	D	E	d	133	104	-87	-86	142		F	G	H	I	J	d	101	185	167	209	97	M1		Differences (consistent with H_1) At least 5 correct differences
	A	B	C	D	E																											
d	133	104	-87	-86	142																											
	F	G	H	I	J																											
d	101	185	167	209	97																											
	$\bar{d} = 96.5$ $s = 103.2$ $n = 10$ $t = \frac{96.5 - 0}{103.2/\sqrt{10}} = 2.96$					m1dep		Attempt to evaluate \bar{d} and s																								
	$t = \frac{96.5 - 0}{103.2/\sqrt{10}} = 2.96$					B1		Use of $\frac{s}{\sqrt{n}}$ ft in ts $n = 9$ or 10																								
	$df = 9$ $cv = 2.821$ $2.96 > 2.821$					m1		Method for t																								
	$df = 9$ $cv = 2.821$ $2.96 > 2.821$					A1		$(\pm) 2.96 (2.85 - 3.05)$ or $p=0.008$																								
	Reject H_0 Significant evidence to suggest Robusta produces a higher mean/average yield of green beans. Or Scientist's belief supported by given data					B1		for correct cv or $p = 0.00803 < 0.01$ gains M1m1B1m1A1B1																								
						E1dep		correct conclusion in context 1 tail																								
							8																									

Q2	Solution	Marks	Total	Comments																												
(a)(i)	Assume that differences in pulse/rate measurements are normally distributed and that the 12 people can be regarded as a random sample.	E1 E1	2	Normal dist and random sample in context																												
(ii)	H_0 pop mean diff/ $\mu_d = 0$ H_1 pop mean diff/ $\mu_d \neq 0$ 2 tail 5% $d = \text{After} - \text{before}$ <table border="1"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr><td>d</td><td>5</td><td>6</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr> <tr><td></td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr> <tr><td>d</td><td>5</td><td>1</td><td>-1</td><td>-5</td><td>4</td><td>2</td></tr> </table> $\bar{d} = 1.417 \quad s = 3.175 \quad n = 12$ $t = \frac{1.417 - 0}{\frac{3.173}{\sqrt{12}}} = 1.545$ $df = 11 \quad cv = \pm 2.201$ $1.545 < 2.201$ Accept H_0 No significant evidence to suggest that there is a difference in average pulse rates before and immediately after minor dental treatment		1	2	3	4	5	6	d	5	6	1	0	-1	0		7	8	9	10	11	12	d	5	1	-1	-5	4	2	B1 M1 m1 dep diffs M1 A1 B1 A1 E1	8	Hypotheses sc 0's ignored B1 M1 m1 M1 only Differences – can be reverse sign attempt to find \bar{d} , s seen/labelled Use of $\frac{s}{\sqrt{12}}$ ft on 's' $(\pm) 1.54$ or 1.55 ($1.53 - 1.56$) (or $p = 0.15$ compared with 0.025) for correct cv Correct conclusion with consistent comparison Correct conclusion in context
	1	2	3	4	5	6																										
d	5	6	1	0	-1	0																										
	7	8	9	10	11	12																										
d	5	1	-1	-5	4	2																										
(iii)	Type II error	B1	1																													

Q2	Solution	Marks	Total	Comments																												
(a)	<p>H_0 pop mean diff or $\mu_d = 0$ H_1 pop mean diff or $\mu_d > 0$ [QC - DD] 1 tail 5%</p> <p>$d = QC - DD$</p> <table border="1"> <tr> <td></td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> <td>F</td> </tr> <tr> <td>d</td> <td>+5</td> <td>0</td> <td>+4</td> <td>+3</td> <td>-1</td> <td>-1</td> </tr> <tr> <td></td> <td>G</td> <td>H</td> <td>I</td> <td>J</td> <td>K</td> <td>L</td> </tr> <tr> <td>d</td> <td>+4</td> <td>+2</td> <td>+8</td> <td>+4</td> <td>+9</td> <td>+2</td> </tr> </table> <p>$\bar{d} = 3.25$ $s = 3.166$ $n = 12$</p> <p>$t = \frac{3.25 - 0}{3.166 / \sqrt{12}} = 3.56$</p> <p>$df = 11$ $cv = 1.796$ $1.796 < 3.56$</p> <p>Reject H_0</p> <p>Significant evidence to suggest that: the belief is correct OR the time taken, after placing on ward, before a box of <i>Chok Delights</i> is opened is, on average, less than that for a box of <i>Quality Chox</i></p>		A	B	C	D	E	F	d	+5	0	+4	+3	-1	-1		G	H	I	J	K	L	d	+4	+2	+8	+4	+9	+2	B1 M1 A1 m1dep B1 m1 A1 B1 A1 dep E1dep	10	Hypotheses consistent with d Differences (ignore signs) At least 8 correct differences PI Attempt to evaluate \bar{d} and s sc 0 excluded gives $\bar{d} = 3.545$ M1A1ml max Use of $\frac{s}{\sqrt{n}}$, $n = 12$ ft in ts oe PI Method for t $(\pm) 3.56 (3.50 - 3.60)$ for correct cv Allow p value = 0.00225(for m1 B1 m1A1) compared with 0.05(B1) correct conclusion correct conclusion in context
	A	B	C	D	E	F																										
d	+5	0	+4	+3	-1	-1																										
	G	H	I	J	K	L																										
d	+4	+2	+8	+4	+9	+2																										

Q	Solution	Marks	Total	Comments
1(a)	16 birds/feathers can be regarded as a random sample.	E1	2	At least one in context for E2
	Differences in yellowness index can be assumed to be normally distributed.	E1		sc B1 for both reasons, no context
1(b)	H_0 pop mean diff $\mu_d = 0$	B1		
	H_1 pop mean diff $\mu_d > 0$			
	1 tail 1%			
	$d = \text{Typical} - \text{Odd}$			

Q	Solution	Marks	Total	Comments																								
1(a)(i)	H_0 pop mean diff $\mu_d = 0$ H_1 pop mean diff $\mu_d \neq 0$ 2 tail 5%	B1		Must refer to pop mean differences or μ_d																								
	$d = K - EMC$ <table border="1" data-bbox="258 384 711 518"> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> <tr> <td>d</td> <td>-0.4</td> <td>0.8</td> <td>-0.4</td> <td>-0.5</td> <td>0.1</td> </tr> <tr> <td></td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td></td> <td>-0.8</td> <td>-0.5</td> <td>0</td> <td>-0.6</td> <td>0.1</td> </tr> </table>		1	2	3	4	5	d	-0.4	0.8	-0.4	-0.5	0.1		6	7	8	9	10		-0.8	-0.5	0	-0.6	0.1	M1		Differences – can be reversed
	1	2	3	4	5																							
d	-0.4	0.8	-0.4	-0.5	0.1																							
	6	7	8	9	10																							
	-0.8	-0.5	0	-0.6	0.1																							
	$\bar{d} = -0.22$ $s = 0.471$ $n = 10$	m1		attempt to find \bar{d} , s can be implied																								
	$t = \frac{-0.22 - 0}{\frac{0.47}{\sqrt{10}}} = -1.48$	m1 m1 A1		Use of $\frac{s}{\sqrt{10}}$ ft Method for t (\pm) 1.48 (1.46 – 1.48)																								
	$df = 9$ $cv = -2.262$ $-2.262 < -1.48$	B1		for correct cv (or $p = 0.17 > 0.05$ B1)																								
	Accept H_0 <u>No significant evidence</u> to suggest that there is a difference in mean <u>measurements</u> for the two <u>devices</u> .	E1	8	*sc4 '0' ignored scores B1 M1m1m1 $\frac{s}{\sqrt{9}}$ correct conclusion in context																								
(ii)	Assumed that differences in first ray foot measurements are normally distributed	E1	2	Normal distribution mentioned <u>in a sentence</u> E1 Differences in foot measurements are normal gains other E1																								

Q	Solution	Marks	Total	Comments																								
3(a)(i)	H_0 pop mean diff $\mu_d = 5$ H_1 pop mean diff $\mu_d > 5$ 1 tail 10% $d = \text{Brand} - \text{Sup Own}$ <table border="1" data-bbox="255 451 711 586"> <tr> <td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>d</td><td>+7</td><td>+13</td><td>-1</td><td>+6</td><td>-4</td></tr> <tr> <td></td><td>6</td><td>7</td><td>8</td><td></td><td></td></tr> <tr> <td>d</td><td>+16</td><td>+1</td><td>+5</td><td></td><td></td></tr> </table>		1	2	3	4	5	d	+7	+13	-1	+6	-4		6	7	8			d	+16	+1	+5			B1		H_0 pop mean diff $\mu_d = 5$ H_1 pop mean diff $\mu_d < 5$ H_1 must be consistent with d Differences (can be reversed)
	1	2	3	4	5																							
d	+7	+13	-1	+6	-4																							
	6	7	8																									
d	+16	+1	+5																									
	$\bar{d} = 5.375$ $s = 6.781$ $n = 8$ $t = \frac{5.375 - 5}{6.781 / \sqrt{8}} = 0.156$	M1	m1	Attempt to find \bar{d} , s can be implied by correct ts Use of $\frac{s}{\sqrt{8}}$ ft																								
	$df = 7$ $ cv = 1.415$ $0.156 < 1.415$ Accept H_0 Significant evidence to suggest that claim is correct.	M1 m1 A1	B1	Use of $\bar{x} - 5$ or $\bar{x} + 5$ Whole method for t $(\pm)0.156$ (0.15–0.16) SC $t = -4.3275$ B3 For correct consistent cv CAO																								
(ii)	Assumed that differences in jar jam contents are normally distributed and samples of jars are obtained at random.	E1	9	Clear correct conclusion in context of claim or jam contents																								
		E1	2	Diffs of contents / jam normal																								
		E1		Jars obtained at random																								