A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing
Two Sample Mean

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **17** questions in this question paper. The total mark for this paper is ***125***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_SS05_JUNE_2018_3b

Two brands of baking parchment, Brand A and Brand B, are sold in rolls that have a nominal length of 30 metres.

The lengths, in metres, of a random sample of 10 rolls of Brand A baking parchment were:

31.6 29.6 30.3 28.9 29.7 30.5 31.1 28.7 29.1 30.5

Lengths of baking parchment on rolls may be assumed to be normally distributed.

A sample of 12 rolls of Brand B baking parchment yielded a mean length of 30.9 metres and a standard deviation of 1.21 metres.

(i) Show that the value of the pooled estimate of the population variance for the length in metres of rolls of baking parchment is 1.22, correct to three significant figures.

[2 marks]

(ii) Test at the 10% level of significance, whether rolls of baking parchment of Brand B are, on average, more than 20 centimetres longer than those of Brand A.

[8 marks]

AQA_SS05_JUNE_2016_6b

Stephen, another student at the college, read an article in a newspaper stating that male adults who travel to work by car have, on average, a BMI more than 1 kg/m2 greater than those who travel to work by alternative means of transport.

To investigate whether this statement also applies to male students travelling to the college, Stephen calculated the BMI of each student in two independent random samples of male students. The 11 students in one sample travelled to the college by car, whereas the 8 students in the other sample travelled to the college by alternative means of transport. His results are given in the table.

BMI of those travelling by car	28.4	26.7	25.2	27.8	22.8	28.9	23.1	26.7	24.1	27.6	25.8
BMI of those travelling by alternative means	26.7	23.9	23.0	22.5	25.7	26.3	22.6	23.1			

Making any necessary assumptions, investigate, at the 5% significance level, the hypothesis that male students who travel to the college by car have a BMI which is, on average, more than 1 kg/m2 greater than those who travel to the college by alternative means of transport.

[11 marks]

AQA_SS05_JUNE_2017_5b/c

Emily, a midwife, is investigating the birth weights of babies. She suspects that children born in the summer months of June, July and August are, on average, heavier than those born at other times of the year.

She records the weights, x kilograms, of a random sample of six babies she delivered in June, July and August and compares them with the weights, y kilograms, of a random sample of nine babies she delivered at other times of the year. Her results are shown in the table.

x	4.27	3.36	3.18	4.74	4.01	3.72			
y	3.63	4.38	2.85	3.92	3.12	2.77	4.04	4.17	2.54

You may assume that x and y are values from independent normal populations

(b) Investigate Emily's suspicion at the 10% level of significance.

[9 marks]

(c) A Type I or a Type II error may have been made in part (b).

(i) Which type of error might have been made?

(ii) Explain, in context, the meaning of the type of error that you have identified in part (c)(i).

[3 marks]

AQA_SS05_JUNE_2015_2

Brad works as a bird conservationist during the summer months. One of his tasks each summer is to randomly select a number, n, of fledgling puffin chicks just before they leave the nesting site and to record the weight, x grams, of each selected chick. Brad's summarised results for each of the years 2010 and 2012 are shown in the table.

	n	$\sum \mathbf{x}$	$\sum(\boldsymbol{x} - \bar{\boldsymbol{x}})^2$
2010	75	19 230	49 512
2012	90	23 760	49 631

(a) For each year, calculate the mean and the variance of the weights of the chicks selected.

[4 marks]

(b) (i) Use a z-test to investigate Brad's claim that the mean weight of chicks was greater in 2012 than in 2010. Use the 5% significance level.

[6 marks]

(ii) Explain why no distributional assumptions were necessary in carrying out the test in part (b)(i).

[2 marks]

(c) State, in the context of this question, the meaning of a Type I error.

[2 marks]

AQA_SS05_JUNE_2014_2

Jamila, a zoology student, is researching the common toad.

She decides to carry out a hypothesis test to investigate whether the mean length of female common toads is more than 1.5 cm greater than the mean length of male common toads.

(a) State null and alternative hypotheses for her test.

[2 marks]

(b) Jamila collects a sample of 8 female common toads and calculates their mean length to be 8.54 cm. She also collects, independently, a sample of 10 male common toads and calculates their mean length to be 6.28 cm.

Assume that each sample is random and that the lengths of female common toads and male common toads are each normally distributed with a standard deviation of 0.6 cm.

Carry out Jamila's test using the 1% significance level.

[6 marks]

AQA_SS05_JUNE_2013_2

Nasreen owns a small supermarket. She sells boxes of six free-range eggs from each of two local suppliers, Alaric and Belinda.

Nasreen weighs each box in a random sample of boxes of eggs from each supplier. Her results, in grams, are as follows:

Alaric	473	485	474	471	470	463		
Belinda	423	426	432	449	447	438	445	446

The weight of a box of eggs from Alaric may be modelled by a normal distribution with a standard deviation of 7 grams. The weight of a box of eggs from Belinda may be modelled by a normal distribution with a standard deviation of 10 grams.

Investigate, at the 5% level of significance, Nasreen's belief that the mean weight of a box of eggs from Alaric is more than 24 grams greater than the mean weight of a box of eggs from Belinda.

(9 marks)

AQA_SS05_JUNE_2012_4aii/b

A charity shop opened six days a week from Monday to Saturday. Bruno, the manager, decided to open on Sundays on a trial basis starting in November 2011. The Saturday takings, in £, for the 10 weeks before the start of Sunday opening and the Saturday and Sunday takings, in £, for the first 7 weeks of Sunday opening are shown.

Week ending	Saturday takings (£)	Sunday takings (£)
28 August	671	–
4 September	582	–
11 September	611	–
18 September	711	–
25 September	695	–
2 October	648	–
9 October	539	–
16 October	696	–
23 October	723	–
30 October	610	–
6 November	567	234
13 November	589	298
20 November	701	312
27 November	542	342
4 December	591	373
11 December	624	390
18 December	725	421

(a) (ii) Using only the data for Saturday takings, test, at the 5% significance level, whether the mean Saturday takings have dropped after the end of October 2011. Assume that the Saturday takings up to the end of October 2011 and those from the start of November 2011 may be treated as random samples from normal distributions with equal variances.

(11 marks)

(b) Bruno estimates that, in order to cover the additional costs incurred by Sunday opening, the mean total takings for Saturday and Sunday of each week from the start of November 2011 would have to exceed the mean Saturday takings up to the end of October 2011 by more than £50. A statistician agrees to carry out an unpaired t-test, using the data in the table on page 10, to examine whether there is significant evidence that this has been achieved.

(i) State the null and alternative hypotheses for this test.

(2 marks)

(ii) The Saturday takings for weeks ending 28 August 2011 to 30 October 2011 constituted one sample for this test. Write down the values of the other sample.

(2 marks)

(iii) The test statistic for this test is 6.15. Using the 1% significance level, complete the test and state your conclusions in context.

(3 marks)

AQA_SS05_JUNE_2011_5aii

A manufacturing firm frequently uses a courier service to transport documents between its factory in Warwickshire and its head office in London.

The firm currently uses ABC couriers, but XYZ couriers claim that they could deliver the documents in a shorter time.

The firm agrees to give XYZ a trial, and to make a permanent change to them if the mean time for XYZ can be shown to be less than that for ABC by more than 10 minutes.

The last 8 times, in minutes, taken by ABC to deliver the documents are shown. The 7 times, in minutes, taken by XYZ during their trial are also shown.

ABC couriers	187	212	193	206	188	204	231	220
XYZ couriers	179	198	186	219	164	200	201	

(a) (ii) Examine whether the mean time for XYZ is less than that for ABC by more than 10 minutes. Use the 10% significance level.

(9 marks)

AQA_SS05_JUNE_2010_2

Simon is investigating two models, A and B, of vacuum flask of the same capacity to decide which is better at keeping liquid hot. To test a flask, he pours freshly boiled water into it and seals the top. Five hours later, he measures the temperature of the water.

(a) Simon carries out this procedure on 7 occasions using flask A and on 8 occasions using flask B. He records the water temperature, x^0C for flask A and y^0C for flask B, after each test, with the following results.

x :	72.5	76.5	79.5	79.0	78.0	78.5	74.5	
y :	72.0	75.0	74.5	72.0	71.5	73.5	69.5	76.5

Carry out a hypothesis test, at the 1% significance level, to investigate whether there is a difference in the mean water temperature after five hours between flasks A and B. Assume that the given data are random samples from normal distributions with equal variances.

(12 marks)

(b) Simon carried out his tests at home, using kitchen equipment, over a period of 15 days. He used flask A on the first 7 days and flask B on the last 8 days.

Suggest two reasons why his results may not be reliable.

(2 marks)

AQA_SS05_JUNE_2008_3b

Investigation of a number of accidents that had occurred on a fairly quiet stretch of country road suggested that excessive speed had been an important factor. Consequently, a speed camera was installed on this stretch of road. The speeds, in mph, of random samples of vehicles before and after the installation of the speed camera were recorded.

Before speed camera installed	61.4	70.5	78.2	58.5	66.3	84.6	69.4
After speed camera installed	47.2	59.4	62.5	38.3	68.6	58.4	

(b) Test, at the 5% significance level, whether the installation of the speed camera has reduced the average speed of vehicles on this stretch of road.

(9 marks)

AQA_SS05_JUNE_2009_5b

Fidel owns a shop selling fishing tackle. He obtains fishing line from Raoul, a manufacturer. Raoul states that his standard fishing line has a mean breaking strength of 15.3 kg with a standard deviation of 0.65 kg.

Fidel would also like to stock stronger fishing line which he could sell at a higher price. Raoul states that he is able to supply premium fishing line with a mean breaking strength at least 5 kg greater than the standard fishing line. However, the standard deviation would also be increased to 0.95 kg.

Fidel decides to measure the breaking strengths of samples of each type of fishing line with the following results, in kg.

Standard fishing line	15.9	16.4	14.8	15.2	14.3	14.9	15.0
Premium fishing line	18.8	20.4	22.1	19.1	19.3	18.7	

(b) Assuming that the standard deviations of the breaking strengths of the two types of fishing line are as stated by Raoul, test whether the data are consistent with the mean breaking strength of the premium fishing line being at least 5 kg greater than the mean breaking strength of the standard fishing line. Use the 10% significance level.

(7 marks)

AQA_SS05_JUNE_2007_3b

Sandeep is training for a marathon. Each weekday he runs the same route from his home in the morning and again in the evening. He records his time for each run.

For random samples of 8 morning runs and 10 evening runs, his times, in minutes, are as follows:

Morning:	64.1	69.2	62.1	70.2	65.9	71.8	63.7	64.9		
Evening:	62.8	60.9	65.8	77.6	66.3	75.2	72.7	74.2	65.3	63.5

(b) On Saturdays, Sandeep trains at an athletics stadium. He completes a set number of laps on the running track in the morning and the same number of laps in the afternoon. His running times for both morning and afternoon are normally distributed, each with a standard deviation of 2.1 minutes.

For a random sample of 12 morning runs, Sandeep's mean running time is 61.7 minutes.

For a random sample of 9 afternoon runs, his mean running time is 58.9 minutes.

Sandeep's trainer claims that, on average, Sandeep completes the run more than one minute faster in the afternoon than in the morning. Carry out a hypothesis test, at the 5% significance level, to determine whether the evidence supports this claim.

(7 marks)