Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2015

Statistics SS05

Unit Statistics 5

Tuesday 16 June 2015 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The lifetime, X, of a *LastLong* tyre, measured in tens of thousands of miles, can be modelled by an exponential distribution with probability density function

$$f(x) = \begin{cases} \frac{1}{3}e^{-\frac{x}{3}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) For a *LastLong* tyre, state the value of:

(i) the mean lifetime;

(ii) the standard deviation of the lifetime.

[2 marks]

(b) Find the probability that the lifetime of a randomly selected *LastLong* tyre is:

(i) less than $10\,000$ miles;

(ii) between $30\,000$ and $40\,000$ miles.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 Brad works as a bird conservationist during the summer months. One of his tasks each summer is to randomly select a number, n, of fledgling puffin chicks just before they leave the nesting site and to record the weight, x grams, of each selected chick. Brad's summarised results for each of the years 2010 and 2012 are shown in the table.

	n	$\sum x$	$\sum (x - \overline{x})^2$
2010	75	19 230	49 512
2012	90	23 760	49 631

(a) For **each** year, calculate the mean and the variance of the weights of the chicks selected.

[4 marks]

(b) (i) Use a z-test to investigate Brad's claim that the mean weight of chicks was greater in 2012 than in 2010. Use the 5% significance level.

[6 marks]

(ii) Explain why no distributional assumptions were necessary in carrying out the test in part **(b)(i)**.

[2 marks]

(c) State, in the context of this question, the meaning of a Type I error.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 2

3 Dhiraj currently lives with his parents outside Darlington. He decides to seek his own accommodation either in Darlington or by relocating to London.

Using an internet site, he selects a number of one-bedroom unfurnished flats in Darlington and records the rental cost per calendar month, £x, of each flat selected.

Using the same internet site, he also selects a number of one-bedroom unfurnished flats in the area of London in which he would like to live and records the rental cost per calendar month, £y, of each flat selected. His results are shown in the table.

x	550	525	525	350	485	450	495	295	500			
y	1500	1400	1384	1860	1450	1575	1492	1680	1820	1625	1600	1595

You may assume that x and y are values from independent populations with variances ${\sigma_x}^2$ and ${\sigma_y}^2$ respectively.

(a) (i) Investigate, at the 5% significance level, the hypothesis that ${\sigma_x}^2 = {\sigma_y}^2$.

[7 marks]

(ii) State **two** further assumptions that need to be made for the test in part **(a)(i)** to be valid.

[2 marks]

(b) Dhiraj believes that the rental cost of a one-bedroom unfurnished flat is, on average, more than £1000 per calendar month greater in London than in Darlington. Dhiraj decides to use a hypothesis test to investigate his belief.

(i) Write down suitable null and alternative hypotheses for Dhiraj's test.

(ii) Assuming that all necessary assumptions are satisfied, complete Dhiraj's hypothesis test using the 5% significance level.

[10 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

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Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 Sandy is a keen cyclist who buys a new racing bicycle to compete in 15-km races. A sample of 14 of Sandy's race times, in minutes, using the new bicycle gave the following results.

24.2 23.7 26.1 23.8 22.6 24.9 22.7 27.3 25.9 26.4 23.1 25.0 25.2 24.5

Sandy's coach wishes to use this sample to construct confidence intervals for both the population variance and the population mean of Sandy's 15-km race times.

(a) (i) What assumption, other than that the sample of Sandy's 15-km race times is random, must be made for these confidence intervals to be valid?

[1 mark]

(ii) Construct a 95% confidence interval for the population variance of Sandy's 15-km race times.

[7 marks]

(iii) Construct a 95% confidence interval for the population mean of Sandy's 15-km race times.

[6 marks]

(b) When using an older bicycle, Sandy's 15-km race times, in minutes, had a mean of 26.2 and a variance of 3.39.

With reference to the confidence intervals you have calculated in parts **(a)(ii)** and **(a)(iii)**, comment on Sandy's performance in 15-km races when using the new bicycle.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

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Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

5 Alex buys a jewellery-making kit containing beads and threads.

(a) There are seven colours of beads: red, yellow, pink, green, orange, purple and blue.

Each kit contains a random sample of exactly 150 of these beads.

The list of contents included in each kit claims that the beads are produced with colours in the ratios $5:4:3:2:4:2:5$ respectively.

Alex opens the kit and counts the number of beads of each of the seven colours. The results are given in the table.

Colour	**Red**	**Yellow**	**Pink**	**Green**	**Orange**	**Purple**	**Blue**
Number of beads	37	22	11	15	25	9	31

Carry out a χ^2 goodness of fit test at the 1% level of significance to investigate whether the ratios of the colours of beads in the kit bought by Alex provide evidence to support the claim stated in the list of contents.

[8 marks]

QUESTION PART REFERENCE

Answer space for question 5(a)

QUESTION PART REFERENCE

Answer space for question 5(a)

Question 5 continues on the next page

5 (b) Each jewellery-making kit contains exactly 10 threads. The lengths, x centimetres, of the threads in a kit may be modelled by a rectangular distribution with probability density function

$$\mathrm{f}(x) = \begin{cases} k & 37.5 < x < 47.5 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(i) State the value of k.

[1 mark]

(ii) Find values for the mean and the standard deviation of X.

[3 marks]

(iii) Calculate the probability that a randomly selected thread will have a length within one standard deviation of the mean.

[2 marks]

(iv) Alex wishes to make an arrangement of beads for a necklace that requires a thread with a length greater than $45\,\mathrm{cm}$.

Given that the lengths of threads in the kit are independent of each other, calculate the probability that the kit bought by Alex contains at least one such thread.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5(b)

QUESTION PART REFERENCE

Answer space for question 5(b)

QUESTION PART REFERENCE

Answer space for question 5(b)

END OF QUESTIONS