

A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing

Two Way ANOVA

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions **on paper**
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- **You may use the** booklet 'Statistical Formulae and Tables'
- There are **8** questions in this question paper. The total mark for this paper is **49**
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

AQA_JUNE_2014_5

| Q | Solution | Marks | Total | Comments | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------------|--|--|-------|---|----|---------|-------|---|-------|------------|--------|---|-------|-------|-------|----|------|-------|--------|----|----|---|---|---|--|---|----|---|---|---|----|---|---|---|--|---|---|---|---|---|---|---|---|---|
| 5(a)(i) | $A \quad B \quad C \quad D \quad E \quad F \quad G$ $T_A=13 \quad T_B=33 \quad T_C=45 \quad T_D=16 \quad T_E=18 \quad T_F=21 \quad T_G=17$ $n_A=3 \quad n_B=3 \quad \dots \quad n_G=3$ $T = 163$ $\sum \sum x_{ij}^2 = 1705 \quad N = 21$ Makings $\sum \frac{T_j^2}{n_j} = \frac{73^2}{7} + \frac{54^2}{7} + \frac{36^2}{7} = 1363$ Tea Brands $\sum \frac{T_i^2}{n_i} = \frac{13^2}{3} + \frac{33^2}{3} + \dots + \frac{17^2}{3} = 1531$ $SS_{\text{makings}} = 1363 - \frac{163^2}{21} = 97.81$ $SS_{\text{teas}} = 1531 - \frac{163^2}{21} = 265.81$ $SS_{\text{Total}} = 1705 - \frac{163^2}{21} = 439.81$ <table border="1"> <thead> <tr> <th></th> <th>SS</th> <th>df</th> <th>ms</th> </tr> </thead> <tbody> <tr> <td>Makings</td> <td>97.81</td> <td>2</td> <td>48.91</td> </tr> <tr> <td>Tea Brands</td> <td>265.81</td> <td>6</td> <td>44.30</td> </tr> <tr> <td>Error</td> <td>76.19</td> <td>12</td> <td>6.35</td> </tr> <tr> <td>Total</td> <td>439.81</td> <td>20</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> | | SS | df | ms | Makings | 97.81 | 2 | 48.91 | Tea Brands | 265.81 | 6 | 44.30 | Error | 76.19 | 12 | 6.35 | Total | 439.81 | 20 | | | | | | | | | | | | | | | | | | | | | | | | |
| | SS | df | ms | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Makings | 97.81 | 2 | 48.91 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Tea Brands | 265.81 | 6 | 44.30 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Error | 76.19 | 12 | 6.35 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Total | 439.81 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | M1ft | | SS for makings attempt | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | M1ft | | SS for teas attempt | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | M1ft | | SS for total attempt | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | M1ft | | Error SS ft (not -ve) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | m1ft | | Method for ms (not -ve) Condone only one correct or clear method seen ft | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $H_0: \mu_{\text{first}} = \mu_{\text{second}} = \mu_{\text{third}}$ $H_1: \text{at least 2 of the means differ}$ $T_s = \frac{48.91}{6.35} = 7.70 \quad F^2_{12} = 3.885 < 7.70$ Reject H_0. $H_0: \mu_A = \mu_B = \dots = \mu_G$ $H_1: \text{at least 2 of the means differ}$ $= \frac{44.30}{6.35} = 6.98 \quad F^2_{12} = 2.996 < 6.98$ Reject H_0. | B1 m1ft B1 A1dep | | Correct hypotheses seen once Method for F for makings cv correct 3.885 or $p = 0.007$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | m1ft B1 (A1) | | A1 dep ts/cv or p correct Method for F for teas cv correct 2.996 or $p = 0.0022$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | There is a significant difference between at least two of the making orders and between at least two of the brands | E1dep | 12 | A1 for Reject for both E1 In context for both dep A1 Might see in earlier conclusions | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (ii) | First making is significantly preferred to the third making. Don't use a tea bag more than twice, preferably only once Brand C seems to be the favourite tea brand and Brand A the least favourite | E1 or E1 or E1 | 2 | For any two relevant comments | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (b)(i) | There is no interaction [between tea brand and making order.] One brand is not better/worse at particular making. | B1 E1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (ii) | The population of ratings is normal and the ratings have a common variance | B1 E1 | 4 | Normal and common variance Reference to context/ratings somewhere or μ_a, η_d used | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (c) | $H_0: \text{pop mean}/\text{median diff} = 0$ $H_1: \text{pop mean}/\text{median diff} \neq 0$ Ranks <table border="1"> <tr> <td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td> </tr> <tr> <td>9</td><td>2</td><td>5</td><td>.</td><td>1</td><td>3½</td><td>7</td><td>6</td><td>8</td><td>3½</td> </tr> <tr> <td>1</td><td>8</td><td>5</td><td></td><td>9</td><td>6½</td><td>3</td><td>4</td><td>2</td><td>6½</td> </tr> <tr> <td>-</td><td>+</td><td>-</td><td></td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>+</td> </tr> </table> $T_- = 9 + 5 + 1 + 3\frac{1}{2} + 7 + 6 + 8 = 39\frac{1}{2}$ $T_+ = 2 + 3\frac{1}{2} = 5\frac{1}{2}$ $ts = T_+ = 5\frac{1}{2} \quad cv = 6 \quad T_+ < 6 \quad \text{Reject } H_0$ There is a significant difference the brands - C preferred | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 9 | 2 | 5 | . | 1 | 3½ | 7 | 6 | 8 | 3½ | 1 | 8 | 5 | | 9 | 6½ | 3 | 4 | 2 | 6½ | - | + | - | | - | - | - | - | - | + | M1 m1 A1 B1 E1dep | 5 | ranks - any effort totals of ranks +/- correct cv correct and correct comparison with lower ts in context ts and cv correct and hypotheses effort |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 2 | 5 | . | 1 | 3½ | 7 | 6 | 8 | 3½ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 8 | 5 | | 9 | 6½ | 3 | 4 | 2 | 6½ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| - | + | - | | - | - | - | - | - | + | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| 5(a)(i) | <table border="1"> <thead> <tr> <th></th><th>O</th><th>M</th><th>R</th><th>Tot</th></tr> </thead> <tbody> <tr> <td>A</td><td>42</td><td>29</td><td>19</td><td>90</td></tr> <tr> <td>B</td><td>37</td><td>33</td><td>24</td><td>94</td></tr> <tr> <td>C</td><td>24</td><td>29</td><td>18</td><td>71</td></tr> <tr> <td>D</td><td>25</td><td>22</td><td>13</td><td>60</td></tr> <tr> <td>Tot</td><td>128</td><td>113</td><td>74</td><td>315</td></tr> </tbody> </table> <p>$\Sigma x^2 = 9019$</p> <p>Between Models SS</p> $\left(\frac{90^2 + 94^2 + 71^2 + 60^2}{3} \right) - \frac{315^2}{12} = 256.92$ <p>Between Campers SS</p> $\left(\frac{128^2 + 113^2 + 74^2}{4} \right) - \frac{315^2}{12} = 388.5$ <p>Total SS = $9019 - \frac{315^2}{12} = 750.25$</p> <table border="1"> <thead> <tr> <th>Source</th><th>SS</th><th>DF</th><th>MS</th></tr> </thead> <tbody> <tr> <td>Models</td><td>256.92</td><td>3</td><td>85.64</td></tr> <tr> <td>Campers</td><td>388.50</td><td>2</td><td>194.25</td></tr> <tr> <td>Error</td><td>104.83</td><td>6</td><td>17.47</td></tr> <tr> <td>Total</td><td>750.25</td><td>11</td><td></td></tr> </tbody> </table> <p>H_0: no difference between models $F = 85.64/17.47 = 4.90$ $cv F_{[3,6]}$ is 4.757 reject H_0 - not all models take same time to pitch on average</p> <p>H_0: no difference between campers $F = 194.25/17.47 = 11.1$ $cv F_{[2,6]}$ is 5.143 reject H_0 - not all campers take same time to pitch a tent on average</p> | | O | M | R | Tot | A | 42 | 29 | 19 | 90 | B | 37 | 33 | 24 | 94 | C | 24 | 29 | 18 | 71 | D | 25 | 22 | 13 | 60 | Tot | 128 | 113 | 74 | 315 | Source | SS | DF | MS | Models | 256.92 | 3 | 85.64 | Campers | 388.50 | 2 | 194.25 | Error | 104.83 | 6 | 17.47 | Total | 750.25 | 11 | | | | | | |
|---------|---|-----|--------|---|---|-----|---|----|----|----|----|---|----|----|----|----|---|----|----|----|----|---|----|----|----|----|-----|-----|-----|----|-----|--------|----|----|----|--------|--------|---|-------|---------|--------|---|--------|-------|--------|---|-------|-------|--------|----|--|--|--|--|--|--|
| | O | M | R | Tot | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 42 | 29 | 19 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B | 37 | 33 | 24 | 94 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | 24 | 29 | 18 | 71 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| D | 25 | 22 | 13 | 60 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Tot | 128 | 113 | 74 | 315 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Source | SS | DF | MS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Models | 256.92 | 3 | 85.64 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Campers | 388.50 | 2 | 194.25 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Error | 104.83 | 6 | 17.47 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Total | 750.25 | 11 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method between models SS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method between campers SS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method total SS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method Error SS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | B1 | df 3,2,6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | ml | MS - their df | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method for F - their positive SS and df | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | B1 | 4.757 and 5.143 (2dp) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | A1✓ | conclusion - must be compared with upper tail of F | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | M1 | method for F - their positive SS and df | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | A1 | 4.90 (4.85 - 4.95) and 11.1(11.0 to 11.2) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | A1✓ | conclusion- must be compared with upper | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | A1✓ | with upper tail of F | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | both conclusions in context – needs both previous A1✓ marks | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Q | Solution | Marks | Total | Comments |
|----------|---|-------|-------|-----------------------------|
| 5(a)(ii) | D appears to take least time to pitch but this could be because it is always pitched last after practice on other models. | B1 | | D CAO |
| | | E1 | | because always pitched last |

| | Solution | Marks | Total | Comments | | | | | | | | | | | | | | | | | | | | |
|-------|---|-------|-----------|----------|----|-----|------|---|------|-------|-------|---|------|-------|------|---|-------|-------|-------|---|--|--|----|--|
| 5(b) | $H_0: \mu_{Run} = \mu_{Cycle} = \mu_{Swim}$ $H_1: \text{at least 2 of the means differ}$ $T_{Male} = 26.0 \quad T_{Female} = 23.6$ $n_{Male} = 3 \quad n_{Female} = 3$ $T_{Run} = 14.7 \quad T_{Cycle} = 17.5 \quad T_{Swim} = 17.4$ $n_{Run} = 2 \quad n_{Cycle} = 2 \quad n_{Swim} = 2$ $T = 49.6 \quad \sum \sum x_{ij}^2 = 413.78 \quad N = 6$ Total SS $413.78 - \frac{49.6^2}{6} = 3.753$ Sex SS $\frac{26.0^2}{3} + \frac{23.6^2}{3} - \frac{49.6^2}{6} = 0.96$ Sport SS $\frac{14.7^2}{2} + \frac{17.5^2}{2} + \frac{17.4^2}{2} - \frac{49.6^2}{6} = 2.523$ <table border="1"> <thead> <tr> <th></th> <th>SS</th> <th>df</th> <th>MS</th> </tr> </thead> <tbody> <tr> <td>Sex</td> <td>0.96</td> <td>1</td> <td>0.96</td> </tr> <tr> <td>Sport</td> <td>2.523</td> <td>2</td> <td>1.26</td> </tr> <tr> <td>Error</td> <td>0.27</td> <td>2</td> <td>0.135</td> </tr> <tr> <td>Total</td> <td>3.753</td> <td>5</td> <td></td> </tr> </tbody> </table> $F = \frac{1.26}{0.135} = 9.35 \quad F^2 = 19.0$ $9.35 < 19 \quad \text{Accept } H_0$ There is no significant evidence of a difference in mean training times for the 3 sports. | | SS | df | MS | Sex | 0.96 | 1 | 0.96 | Sport | 2.523 | 2 | 1.26 | Error | 0.27 | 2 | 0.135 | Total | 3.753 | 5 | | M1 M1 M1 M1 B1 m1 m1 A1 B1 E1 | 10 | Total SS method (can be implied in table) Sex SS method Sport SS method Error SS ft (not -ve) Error df correct $v = 2$ Method for MS ft (dep SSe) Method (dep prev Ms) for F (sex/error or sport/error) Not -ve Sports F correct 9.2–9.5 cv correct CAO or $p = 0.117$ Correct conclusion in context, ft small arithmetic error in F |
| | SS | df | MS | | | | | | | | | | | | | | | | | | | | | |
| Sex | 0.96 | 1 | 0.96 | | | | | | | | | | | | | | | | | | | | | |
| Sport | 2.523 | 2 | 1.26 | | | | | | | | | | | | | | | | | | | | | |
| Error | 0.27 | 2 | 0.135 | | | | | | | | | | | | | | | | | | | | | |
| Total | 3.753 | 5 | | | | | | | | | | | | | | | | | | | | | | |
| | Total | | 22 | | | | | | | | | | | | | | | | | | | | | |

| (b) | <table border="1"> <thead> <tr> <th>source</th><th>SS</th><th>DF</th><th>MS</th></tr> </thead> <tbody> <tr> <td>alcohol</td><td>9348</td><td>2</td><td>4674</td></tr> <tr> <td>weights</td><td>7980</td><td>3</td><td>2660</td></tr> <tr> <td>residual</td><td>3214</td><td>6</td><td>535.67</td></tr> <tr> <td>total</td><td>20542</td><td>11</td><td></td></tr> </tbody> </table> | source | SS | DF | MS | alcohol | 9348 | 2 | 4674 | weights | 7980 | 3 | 2660 | residual | 3214 | 6 | 535.67 | total | 20542 | 11 | | B1 | 2, 3, 6 df method for all MS (including method for residual SS), their df |
|----------|---|--------|--|----|----|---------|------|---|------|---------|------|---|------|----------|------|---|--------|-------|-------|----|--|----|--|
| source | SS | DF | MS | | | | | | | | | | | | | | | | | | | | |
| alcohol | 9348 | 2 | 4674 | | | | | | | | | | | | | | | | | | | | |
| weights | 7980 | 3 | 2660 | | | | | | | | | | | | | | | | | | | | |
| residual | 3214 | 6 | 535.67 | | | | | | | | | | | | | | | | | | | | |
| total | 20542 | 11 | | | | | | | | | | | | | | | | | | | | | |
| | H_0 : no difference between amounts of alcohol | m1 | method for F (either) – their figures | | | | | | | | | | | | | | | | | | | | |
| | $F = \frac{4674}{535.67} = 8.73$ reject H_0 : significant evidence differences in mean times to do Sudoku between groups drinking different amounts of alcohol | A1 | 8.73(8.72~8.73) and 4.97(4.96~4.97) | | | | | | | | | | | | | | | | | | | | |
| | $F = \frac{2660}{535.67} = 4.97$ c.v. $F_{[3,6]} = 4.757$ reject H_0 : significant evidence differences in mean times to do Sudoku between groups of different weights | B1 | 5.143(5.14~5.15) and 4.757(4.75~4.76) | | | | | | | | | | | | | | | | | | | | |
| | A1 \wedge | 6 | both conclusions – their figures – must be compared with upper tail of F | | | | | | | | | | | | | | | | | | | | |