A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing
Paired Sample Wilcoxon Rank Test

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

• Use **black** ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.

• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions ***on paper***

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Unless otherwise stated, statistical tests should be carried out at the 5% significance level.

• When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

• ***You may use the*** booklet 'Statistical Formulae and Tables'

• There are **14** questions in this question paper. The total mark for this paper is ***159***

• The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

• Check your answers if you have time at the end.

AQA_JUNE_2015_6b

(b) A further 10 healthy adult volunteers agreed to take part in a new trial. On a particular Wednesday, each volunteer was randomly assigned to take, half an hour before bedtime, either 20 mg of the new drug or 50 mg of the new drug.

On the following Wednesday, those volunteers who originally took 20 mg of the new drug then took 50 mg and those volunteers who originally took 50 mg of the new drug then took 20 mg.

The time taken, in minutes, to achieve persistent sleep by each of the 10 volunteers on each Wednesday was measured. These results are given in the table.

Adult	A	B	C	D	E	F	G	H	I	J
20 mg drug	3.2	4.3	3.8	4.2	4.1	2.9	3.2	4.8	4.4	3.5
50 mg drug	3.4	2.2	2.9	4.8	2.4	3.9	3.5	2.9	2.2	3.1

(i) Explain why the volunteers were randomly assigned to either the 20 mg or the 50 mg dose of the new drug.

(ii) Carry out a Wilcoxon signed-rank test, at the 5% level of significance, to investigate whether the average number of minutes taken by healthy adults to achieve persistent sleep is lower when taking 50 mg of the new drug half an hour before bedtime than when taking 20 mg of the new drug half an hour before bedtime.

[9 marks]

AQA_JAN_2013_3

A chef decides to investigate whether the taste of food is influenced by the sounds heard whilst eating it. She selects 8 regular customers at her restaurant and asks them if they would each be willing to eat a free seafood dish on two separate occasions and score each dish on a scale of 0 to 20 for taste.

The seafood dishes are identical, but the background sounds played through the restaurant's speakers differ on the two occasions. Seaside sounds are played on one of the occasions when the seafood dish is eaten, but the usual background music is played on the other occasion when the seafood dish is eaten.

The scores are given in the table.

	Customer							
	A	B	C	D	E	F	G	H
Score with seaside sounds playing	17	15	14	9	16	19	11	10
Score with usual background music playing	13	9	12	12	17	16	6	3

Carry out a Wilcoxon signed-rank test, using the 5% significance level, to investigate whether, on average, the taste score for the seafood dish is higher when the seaside sounds are played than when the usual background music is played.

(8 marks)

AQA_JAN_2011_2

It is claimed that the lengths of prison sentences for burglary crimes in the UK increased over the ten years from 1994 to 2004.

Ten burglars, who were sentenced in 1994, were selected at random.

(a) These burglars were paired with ten burglars sentenced for similar crimes in 2004.

The length, in months, of the sentence given to each burglar is given in the table.

Burglar pair		A	B	C	D	E	F	G	H	I	J
Year	1994	10	12	15	23	16	19	10	17	24	9
	2004	14	20	14	28	25	12	20	24	30	20

Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate the claim. Interpret your conclusion in context.

(9 marks)

(b) It was suggested that a better design for this research would have been to select the ten burglars sentenced for crimes in 2004 at random. Explain briefly the advantage of using the original design, rather than this new suggested design, for the investigation in part (a).

(2 marks)

AQA_JUNE_2012_4

Two different skills tests, Test A and Test B, are used regularly to evaluate the suitability of applicants for employment as machine operators at a large engineering company. The company's director wishes to compare Test A and Test B.

Each test comprises 10 different tasks, and it is decided that the 10 tasks from each test will be combined in a random order to produce one combined test comprising all 20 tasks.

This combined test is given to nine applicants. For each applicant, the scores for the two separate sets of 10 tasks are totalled to give a score for those tasks originally from Test A and a score for those tasks originally from Test B.

These scores, as percentages, are given in the table.

Applicant	1	2	3	4	5	6	7	8	9
Score									
Test A	48	73	65	53	23	78	47	91	49
Test B	44	74	82	64	38	81	49	81	62

The nine applicants may be regarded as a random sample.

(a) Calculate values for the mean and the standard deviation of the scores for:

(i) Test A;

(ii) Test B.

(3 marks)

(b) Find the value of the product moment correlation coefficient between the scores for the two tests.

(3 marks)

(c) (i) Carry out a Wilcoxon signed-rank test, at the 5% level of significance, to investigate whether there is any difference in the mean scores for the two tests.

(8 marks)

(ii) State the assumption, regarding the distribution of scores, that was necessary for the test in part (c)(i) to be valid.

(1 mark)

(d) With reference to your findings in parts (a), (b) and (c)(i), compare Test A and Test B.

(3 marks)

(e) Dexter, the Head of Recruitment for the engineering company, felt that the suggested combined skills test was too long for applicants. He suggested instead that Test A and Test B should be given to two separate groups of applicants and then the scores of the group taking Test A could be compared with the scores of the group taking Test B.

Give two reasons why using the combined skills test was preferable to Dexter's suggestion when comparing Test A and Test B.

(2 marks)

AQA_JUNE_2013_2

As part of an investigation into trends in local authority spending in England, one of the categories of expenditure considered was 'Highways'. For a random sample of 10 local authorities, the percentages of their total expenditure on 'Highways' during 2002 and also during 2012 are shown in the table.

Local authority		A	B	C	D	E	F	G	H	I	J
Year	2002	5.6	8.4	8.7	9.3	9.9	9.4	7.9	8.1	9.6	8.6
	2012	8.9	8.4	7.9	8.4	10.2	10.1	8.3	9.8	9.5	9.7

(a) Carry out a Wilcoxon signed-rank test to investigate whether there was a change in the average percentage of local authority total expenditure on 'Highways' between 2002 and 2012. Use the 5% level of significance.

(9 marks)

(b) (i) State the assumption necessary regarding the distribution of differences in percentage expenditures in order for the test in part (a) to be valid.

(ii) Suggest an alternative test that could be used for the investigation in part (a) if the assumption in part (b)(i) was known to be invalid.

(2 marks)

(c) For a Wilcoxon signed-rank test carried out on 10 matched pairs, find: (i) the minimum value possible for the test statistic T ; (ii) the maximum value possible for the test statistic T

(3 marks)

AQA_JUNE_2014_3

An investigation was carried out into the effectiveness of two well-known drugs, A and B, in relieving pain for adult arthritis sufferers. Twelve adult arthritis sufferers each volunteered to record the number of hours of relief from pain gained when taking drug A and when taking drug B. The adults agreed to take one of the drugs, A or B, on one day and the other drug on another day. The order of taking the drugs was randomly assigned.

The recorded times, in hours, are given in the table.

Adult	1	2	3	4	5	6	7	8	9	10	11	12
Drug A	2.0	3.6	2.6	2.6	7.2	3.4	6.5	2.4	11.5	2.5	6.1	8.5
Drug B	3.5	5.7	2.8	2.4	9.8	3.3	5.9	4.9	13.5	3.7	9.1	12.4

(a) Explain the purpose of randomly assigning the order of taking drug A or drug B.

[2 marks]

(b) Carry out a Wilcoxon signed-rank test, at the 2% significance level, to investigate whether or not there is a difference between the average number of hours of relief from pain gained by arthritis sufferers when taking drug A and that when taking drug B.

[10 marks]

(c) Give a reason why your conclusion in part (b) might not apply to all adult arthritis sufferers.

[1 mark]

AQA_JUNE_2016_1

An investigation into the effects of caffeine consumption on coordination and processing skills was carried out by a drinks company. Ten university students, who normally consumed fewer than three caffeinated drinks per week, agreed to participate in this investigation. The students agreed to perform several tasks involving hand-eye coordination on two separate occasions.

Students were randomly assigned to have either a caffeinated drink or an uncaffeinated drink thirty minutes before they performed the tasks on the first occasion. One week later they returned to perform the tasks again. Thirty minutes before the students performed the tasks on the second occasion, those who were given a caffeinated drink on the first occasion were given an uncaffeinated drink and those who were given an uncaffeinated drink on the first occasion were given a caffeinated drink.

The scores gained by each student for the hand-eye coordination tasks were on a scale of 0 to 50, where 50 indicates the highest level of hand-eye coordination. The scores for each student on each occasion are given in the table.

Student	A	B	C	D	E	F	G	H	I	J
Caffeinated drink	28	39	38	40	41	26	36	37	22	33
Uncaffeinated drink	32	30	34	37	42	21	34	29	21	27

(a) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether the consumption of a caffeinated drink has any effect on the median score gained in the hand-eye coordination tasks.

[9 marks]

(b) (i) Write down two assumptions that must be made in order for the test in part (a) to be valid.
(ii) Explain, in the context of this question, the meaning of a Type I error.

[4 marks]

AQA_JUNE_2017_1

Researchers at a university department of medicine carried out an investigation into the immediate effect of smoking a cigarette on systolic blood pressure. The researchers selected 10 male regular smokers who were each aged between 20 years and 29 years and had a body mass index (BMI) between 20 and 24

Each man had his systolic blood pressure, x mmHg (millimetres of mercury), measured following three hours of not smoking a cigarette. Then, immediately after smoking a cigarette, each man had his systolic blood pressure, y mmHg, measured again.

The results are given in the table.

Man	1	2	3	4	5	6	7	8	9	10
x	110	117	112	120	119	128	106	109	128	117
y	119	121	120	119	119	123	124	119	125	124

The men may be regarded as a random sample of male regular smokers aged between 20 years and 29 years with a BMI of between 20 and 24

(a) Carry out a Wilcoxon signed-rank test, using the 5% level of significance, to investigate whether, for such men, systolic blood pressure is, on average, lower after three hours of not smoking a cigarette than immediately after smoking a cigarette.

[8 marks]

(b) Give one reason why it would be preferable for researchers to select the 10 regular smokers involved in this research from males only rather than to select them from both males and females.

[2 marks]

AQA_JUNE_2008_1

A manufacturer of an electrical appliance wants to adjust one of the components used in the appliance. The effect that the adjustment would have on the resistance of the component is investigated. The manufacturer selects, at random, 8 components.

Each component has its resistance, in ohms, measured before and after the adjustment.

The results of the investigation are shown in the table.

Component	A	B	C	D	E	F	G	H
Before	38	42	44	35	44	36	44	42
After	41	49	42	40	43	40	46	50

(a) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether or not the average resistance of the component is changed by the adjustment. Interpret your conclusion in context.

(9 marks)

(b) (i) Give one reason why a Wilcoxon signed-rank test might be preferred to a sign test in carrying out an investigation similar to the one carried out in part (a).

(1 mark)

(ii) Describe one situation in which it would not be possible to carry out a Wilcoxon signed-rank test but it would be possible to carry out a sign test.

(2 marks)

AQA_JUNE_2018_2

Ernie, an ecologist, was concerned about the levels of aluminium that accumulate in the wood of poplar trees growing in polluted areas.

A poplar tree was selected in each of 13 different polluted areas and the aluminium content, in micrograms per gram of wood, was measured for each tree in August 2014 and again in November 2014.

Ernie wanted to investigate whether the average aluminium content in poplar trees increased from August 2014 to November 2014.

The results are given in the table. Aluminium content

Area	August 2014	November 2014
1	18.3	12.7
2	13.3	11.1
3	16.5	15.3
4	12.6	12.7
5	9.5	10.5
6	13.6	15.6
7	8.1	11.2
8	8.9	14.2
9	10.0	16.3
10	8.3	15.5
11	7.9	19.9
12	8.1	20.4
13	13.4	36.8

Carry out a Wilcoxon signed-rank test, at the 1% level of significance, to investigate whether the average aluminium content in poplar trees was higher in November 2014 than in August 2014.

[9 marks]

AQA_JAN_2007_6b

Percentage marks were obtained for a random sample of 10 university economics students for their second year examinations in micro-economics and in macro-economics. The results are given in the table.

Student	Micro-economics	Macro-economics
1	38	28
2	41	45
3	47	53
4	51	57
5	54	52
6	56	47
7	59	54
8	61	58
9	63	63
10	70	69

(b) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether the average mark for macro-economics differs from the average mark for micro-economics.

(9 marks)

AQA_JAN_2008_4

National statistics on teenage conceptions were obtained for 1999 and 2003.
The conception rates per 1000 teenagers for a random sample of 10 regions in England and Wales are given in the table.

Region	A	B	C	D	E	F	G	H	I	J
1999	51.1	55.3	48.8	51.0	43.5	49.3	36.4	50.5	35.9	34.1
2003	45.7	52.1	45.0	46.8	41.1	47.2	33.3	50.8	33.1	37.5

(a) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether the average teenage conception rate decreased between 1999 and 2003. Interpret your conclusion in context.

(9 marks)

(b) Explain the advantages of using a matched-pairs design for a test such as the one carried out in part (a).

(2 marks)

(c) Explain, in the context of this question, the meaning of a Type I error.

(2 marks)

AQA_JAN_2010_4b

A survey was carried out during July 2007 to investigate the amount of excise duty, in pence per litre, paid on unleaded petrol and diesel throughout Europe.
The results for a random sample of 10 European countries are given in the table.

Fuel type	Unleaded petrol	Diesel
Country		
Cyprus	24	20
Romania	26	21
Sweden	29	32
Slovakia	33	32
Austria	36	28
Malta	38	26
Finland	47	25
France	48	34
Germany	52	37
UK	57	57

(b) Carry out a Wilcoxon signed-rank test, at the 1% significance level, to investigate whether, in European countries, the average excise duty paid is lower on diesel than on unleaded petrol. Interpret your conclusion in context.

(9 marks)

AQA_JUNE_2007_1

A manufacturer of digital radios seeks the opinions of customers about the performance of its radios before and after introducing a new component.
The manufacturer selects, at random, 10 customers. Each customer is given a radio without the new component and a radio with the new component. Each customer then rates the performance of each radio on a scale from 1 to 20.
The results of the survey are shown in the table.

Customer	A	B	C	D	E	F	G	H	I	J
Without new component	16	18	14	18	15	13	16	8	8	13
With new component	12	16	19	18	11	19	19	15	17	12

Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether customers rated the radio differently after the new component was introduced. Interpret your conclusion in context.

(9 marks)