

A Level Statistics

AQA Past Exam Questions

TOPIC: Hypothesis Testing

One Sample Mean with known Variance

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions **on paper**
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise stated, statistical tests should be carried out at the 5% significance level.
- When a calculator is used, the answer should be given to three significant figures unless otherwise stated.

Information

- **You may use the** booklet 'Statistical Formulae and Tables'
- There are **17** questions in this question paper. The total mark for this paper is **135**
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Check your answers if you have time at the end.

AQA_SS02_JUNE_2015_3

Rika is the manager of a large department store. She believes that the longer their customers stay in the store, the more they are likely to buy. Rika found that customers were spending an average of 24.0 minutes in the store. In an attempt to encourage customers to stay longer, Rika arranged for relaxing music to be played throughout the store. After the music had been introduced, a sample of 120 customers was observed. The mean time spent in the store by these customers was 25.9 minutes, and the standard deviation was 9.5 minutes.

(a) State an assumption that must be made in order to use this sample as a basis for a hypothesis test.

[1 mark]

(b) Assuming that the assumption that you stated in part (a) is valid, test whether the mean time spent in the store by customers has increased. Use the 2% level of significance.

[7 marks]

(c) After the music had been introduced, the mean time spent in the store by customers was, in fact, 25.4 minutes. State, with a reason, whether, in your conclusion in part (b), you made a Type I error, a Type II error or no error.

[2 marks]

AQA_SS02_JAN_2013_2

A machine fills paper bags with flour. Before maintenance on the machine, the weight of the flour in a bag could be modelled by a normal distribution with mean 1005 grams and standard deviation 2.1 grams. Following this maintenance, the flour in each of a random sample of 8 bags was weighed.

The weights, in grams, were as follows.

1006.1 1004.9 1005.8 1007.9 1004.7 1006.3 1007.4 1007.2

(a) Carry out a test, at the 10% significance level, to decide whether the mean weight of flour in a bag filled by the machine had changed. Assume that the distribution of weights was still normal with standard deviation 2.1 grams.

(7 marks)

(b) The flour in each of a random sample of 90 bags was then weighed. For this sample, the mean weight of flour in a bag was 1005.48 grams and the standard deviation was 2.41 grams. Carry out a test, at the 2% significance level, to decide whether the mean weight of flour in a bag filled by the machine had increased from the value of 1005 grams.

(5 marks)

(c) Explain why you did not have to assume that the weight of flour in a bag was normally distributed in order to carry out the test in part (b).

(2 marks)

(d) State, with a reason, which of the tests carried out in parts (a) and (b) might have resulted in a Type II error.

(2 marks)

AQA_SS02_JAN_2012_2

In Blackport, the price of a night's accommodation in a two-star hotel in June 2009 could be modelled by a normal distribution with mean £72.80 and standard deviation £8.70. To investigate whether there had been a change in the mean price, a random sample of 10 two-star hotels advertising on the 'Visit Blackport' website was taken and the corresponding prices, in £, for June 2010 were

72 65 60 80 71 63 54 82 59 65

(a) Show that this sample provides evidence, at the 5% significance level, to conclude that the mean price of two-star hotel accommodation in Blackport had changed. Assume that the distribution of prices in June 2010 was normal with standard deviation £8.70.

(8 marks)

(b) State whether a Type I error or a Type II error might have been made in reaching the conclusion in part (a). Explain your answer.

(2 marks)

(c) State, with a reason, whether the conclusion in part (a) would be valid if you did not know that the prices in June 2010 were normally distributed.

(2 marks)

(d) In addition to the possible errors already considered, state one reason why the conclusion reached in part (a) may not be valid.

(1 mark)

AQA_SS02_JUNE_2016_5

The mean blood cholesterol level of the adult residents of a particular country has been found to be 5.8 millimoles per litre (mmol/l). Monica is a researcher who believes that the daily consumption of yoghurt can reduce blood cholesterol level. She selected a sample of 80 such residents who consumed yoghurt daily and measured the blood cholesterol level, X mmol/l, of each resident, obtaining the following summarised results.

$$\sum x = 452.8 \quad \text{and} \quad \sum(x - \bar{x})^2 = 33.552$$

(a) Show that the results support Monica's belief at the 10% significance level.

[8 marks]

(b) Monica would like to publish the result of her research with the following statement.

"The belief that eating yoghurt daily can reduce blood cholesterol level was supported by my research at the $\alpha\%$ significance level."

State the smallest integer value of α that Monica can use, quoting probabilities to justify your answer.

[2 marks]

(c) Given that the daily consumption of yoghurt actually has no significant effect on blood cholesterol levels, state whether Monica made a Type I error, a Type II error or no error.

[1 mark]

AQA_SS02_JUNE_2013_2

A market trader sells bags of cherries. A sign on his stall says, '1 lb bags of cherries'. The unit of weight 1 lb is equal to 453.6 grams. Sophie, the market inspector, suspects that the bags may, on average, contain less than 453.6 grams.

Sophie asks her assistant, Kevin, to investigate her suspicion. She tells Kevin that, from previous measurements, the weights of bags of cherries from this trader may be assumed to have a normal distribution with standard deviation 10 grams. Kevin weighs 6 bags of cherries and obtains the following weights in grams.

448.2 461.9 455.8 437.0 442.5 441.4

(a) Assuming that the distribution of weights of bags of cherries is still normal with standard deviation 10 grams, investigate Sophie's suspicion at the 10% significance level.

(7 marks)

(b) Sophie discovers that Kevin has included the weight of the bag in his data, as well as the weight of the cherries. Each bag weighs 5 grams. With this information, show that, on the basis of the above data, Sophie can now confirm her suspicion at the 1% significance level.

(4 marks)

(c) Kevin suggests that, in rejecting the trader's claim, Sophie may have made a Type I error or a Type II error. State, giving a reason, which type of error (Type I or Type II) Sophie may have made in this case.

(2 marks)

AQA_SS02_JUNE_2011_5

A company manufactures bath panels. The bath panels should be 700 mm deep, but a small amount of variability is acceptable. The depths are known to be normally distributed with standard deviation 2.1 mm.

(a) In order to check that the mean depth is 700 mm, Amir takes a random sample of 6 bath panels from the current production and measures their depths, in millimetres, with the following results.

701.2 698.2 704.4 699.4 695.5 698.9

Test whether the current mean is 700 mm, using the 5% significance level.

(8 marks)

(b) Isabella, a manager, tells Amir that, in order to check whether the current mean is 700 mm, it is necessary to take a larger sample. Amir therefore takes a random sample of size 40 from the current production and finds that the mean depth is 701.34 mm.

Test whether the current mean is 700 mm, using the data from this second sample and the 5% significance level.

(5 marks)

(c) It is proposed to carry out hypothesis tests at regular intervals to check that the mean remains at 700 mm.

Amir proposes that the tests be based on random samples of size 6, but Isabella favours random samples of size 40. Explain which, if either, sample size would lead to a smaller risk:

- (i) of a Type I error;
- (ii) of a Type II error.

(4 marks)

AQA_SS02_JAN_2011_4

As a special promotion, a supermarket offers cartons of orange juice containing '25% extra' with no increase in price. A random sample of cartons of orange juice was checked. The percentages by which the contents exceeded the nominal quantity were recorded, with the following results.

23.3 27.5 25.7 20.9 24.3 22.6 21.5 22.1

(a) Examine whether the mean percentage by which the contents exceed the nominal quantity is less than 25. Use the 5% significance level. Assume that the data are from a normal distribution with standard deviation 2.3.

(9 marks)

(b) Define a Type I error in the context of this question.

(2 marks)

(c) The supermarket manager claims that, if a larger sample were taken, the risk of making a Type I error would be reduced. Comment on this claim.

(2 marks)

AQA_SS02_JUNE_2010_4

A company manufactures components for fridge motors. The components are designed to have a length of 135.0 millimetres. The lengths, in millimetres, of a random sample of components manufactured on a given Monday were

135.2 135.7 134.8 135.1 136.2 135.7 136.0 135.8 135.5

(a) Examine whether the mean length of components manufactured on that Monday was 135.0 mm. Use the 5% significance level and assume that the lengths of components are normally distributed with a standard deviation of 0.45 mm.

(9 marks)

(b) Explain, in the context of this question, the meaning of a Type I error.

(2 marks)

AQA_SS02_JAN_2010_3

A consumer organisation is investigating the service offered by companies supplying household gas.

(a) The waiting times, in seconds, between a telephone call connecting to a gas company, Northgas, and the caller actually speaking to one of its employees were recorded for nine telephone calls as follows:

76 157 62 56 193 34 89 185 134

Test, using the 5% significance level, whether the mean waiting time for calls made to Northgas exceeds 90 seconds. Assume that this sample is a random sample from a normal distribution with standard deviation 55 seconds.

(8 marks)

(b) Another gas company, Southgas, claims that the waiting time for incoming telephone calls to its offices does not, on average, exceed 90 seconds. A random sample of 85 telephone calls made to Southgas had a mean waiting time of 94 seconds and a standard deviation of 12 seconds. Test the claim made by Southgas using the 5% significance level.

(5 marks)

(c) Angus, a member of the consumer organisation, compared the sample means from parts (a) and (b) and also the conclusions reached. He expressed surprise.

(i) Give a possible reason why Angus expressed surprise.

(2 marks)

(ii) Identify one feature of the samples which explains why, despite his surprise, the conclusions are plausible.

(1 mark)

AQA_SS02_JUNE_2009_5

An ambulance service serves a large rural area. A sample of the times taken, in minutes, from receipt of an emergency call to the arrival of an ambulance at the location of the emergency was recorded as follows.

23 14 27 15 12 26 20 18 22

(a) Assume that these times may be regarded as a random sample from a normal distribution with standard deviation 3.5 minutes. Carry out a hypothesis test, at the 5% significance level, to examine whether the mean time could be equal to 19 minutes.

(8 marks)

(b) It is now decided to examine, at the 5% significance level, whether the mean time is less than 19 minutes.

State the:

- (i) hypotheses;
- (ii) critical value(s);
- (iii) conclusion.

(4 marks)

(c) There is a target of at most 19 minutes for the mean time from receipt of an emergency call to the arrival of an ambulance at the location of the emergency. The director of the ambulance service states that the data provide significant evidence that this target has been achieved.

Comment on this statement.

(3 marks)