1. Tejinder is the deputy manager of a nursery in Gillington. She is interested in the distance parents travel to bring their children to the nursery. She takes a random sample of 40 parents and askes them to fill in a form, stating the distance they travel rom home to the nursery. Her results are summarised in the table below:

Distance (m)	<1	1≤d<2	2≤d<3	3≤d<4	4≤
	16	8	7	4	5

Tejinder decided to use a Chi-squared test to determine whether a normal distribution would provide a suitable model for the distances travelled by customers. Carry out Tejinder's Chi-squared test at the 5% significance level. (Based on S5 2016)

2. Rayson Inc make toilet paper. Gemma, the company statistician, decided to check whether the lengths of Rayson Inc toilet paper are normally distributed and produces the following table

Length (cm)	Frequency
$l < 10.5$	22
$10.5 \leq l < 11$	48
$11 \leq l < 11.5$	53
$11.5 \leq l < 12$	36
$12 \leq l$	19

   a. Obtain estimates of the population mean and standard deviation from the sample
   b. Use the χ^2 distribution, at the 5% significance level, to test whether the normal distribution provides an adequate model for the data.

3. A sample of 120 tea lights are taken from a factory in Roderson. To ensure quality control of the candles' burn time is accurate, it is assumed that said burn time follows an exponential distribution, On behalf of the company, the sample of candles are lit and the time, in minutes, for them to burn out are measured with the following results.

Time (mins)	0-120	121-240	241-300	301-360	361-420	421+
Frequency	3	16	37	32	27	5

Test the hypothesis that the lifetimes of the tea lights follow an exponential distribution.

4. It is claimed that the time it takes for a particular drug to eliminate the symptoms of a 'Hypnic' headache follow an exponential distribution. If so, this would help with making of decisions with regards to distribution of the drug after successfully completing the necessary trials. The table below shows the time declared by a sample of 60 patients from taking the drug and, in their opinion, they symptoms being eliminated.

Time (mins)	0<x≤30	30<x≤60	60<x≤120	120<x≤180	180<x
Frequency	3	19	17	18	3

Use a Chi-squared test to see if the claim of the time taken for the drug to eliminate the symptoms of a 'Hypnic' headache is exponentially distributed are true.

SOLUTIONS 3.3B

1. H_0: the normal distribution will provide a suitable model for the distances travelled by customers.
   H_1: the normal distribution will not provide a suitable model for the distances travelled by customers.

1 tailed	α = 0.05	n = 40	mean = 1.85	s = 1.424

x	upper bound	O	p	E
x < 1	1	16	0.17834	7.1336
1 ≤ x < 2	2	8	0.26667	10.6668
2 ≤ x < 3	3	7	0.24839	9.93356
3 ≤ x < 4	4	4	0.14412	5.7648
4 ≤ x	100	5	0.06554	2.6217
3 ≤ x		9	0.20966	8.3866

v = 1 cv = 3.841

$\sum \frac{(O-E)^2}{E}$ = 12.654

ts > cv
Hence we REJECT H_0
There is sufficient evidence to suggest that the normal distribution will not provide a suitable model for the distances travelled by customers.

2.

   a. Sample mean = 11.199
      S = 0.5897

   b. H_0: the normal distribution provides an adequate model for the data.
      H_1: the normal distribution dos not provide an adequate model for the data.

1 tailed	α = 0.05	n = 178	mean = 11.199	s = 0.5897

x	upper bound	O	p	E
l < 10.5	10.5	22	0.11794	20.99
10.5 ≤ l < 11	11	48	0.24995	44.49
11 ≤ l < 11.5	11.5	53	0.32724	58.25
11.5 ≤ l < 12	12	36	0.217694	38.75
12 ≤ l		19	0.08718	15.52

v = 2 cv = 5.991

$\sum \frac{(O-E)^2}{E}$ = 1.775

ts < cv
Hence we ACCEPT H_0
There is insufficient evidence to suggest that the normal distribution does not provide an adequate model for the data.

3. H_0: the lifetimes of the tea lights follow an exponential distribution.

   H_1: the lifetimes of the tea lights do not follow an exponential distribution.

1 tailed α = 0.05 n = 120 mean = 303.25 λ = 0.003298

P(x ≤ 120) = 0.32791
P(x ≤ 240) = 0.54755
P(x ≤ 300) = 0.62877
P(x ≤ 360) = 0.69541
P(x ≤ 420) = 0.75009

x	upper bound	O	p	E
0-120	120.5	3	0.32791	39.35
121-240	240.5	16	0.21964	26.36
241-300	300.5	37	0.08122	9.75
301-360	360.5	32	0.06664	8
361-420	420.5	27	0.05468	6.56
421+		5	0.24991	29.89

v = 4 cv = 9.488

$\sum \frac{(O-E)^2}{E}$ = 270.23

ts > cv
Hence we REJECT H_0
There is sufficient evidence to suggest that the lifetimes of the tea lights doesn't follow an exponential distribution.

4. H_0: the time taken for the drug to eliminate the symptoms of a 'Hypnic' headache is exponentially distributed
   H_1: the time taken for the drug to eliminate the symptoms of a 'Hypnic' headache is not exponentially distributed

1 tailed α = 0.05 n = 60 mean = 96 λ = 1/96

P(x ≤ 30) = 0.26838
P(x ≤ 60) = 0.46474
P(x ≤ 120) = 0.713495
P(x ≤ 180) = 0.84665

x	upper bound	O	p	E
0<x≤30	30	3	0.26838	16.10
30<x≤60	60	19	0.19636	11.78
60<x≤120	120	17	0.24876	14.93
120<x≤180	180	18	0.13315	7.99
x≤180		3	0.15335	9.20

v = 3 cv = 7.815

$\sum \frac{(O-E)^2}{E}$ = 32.09

ts > cv
Hence we REJECT H_0
There is sufficient evidence to suggest that the time taken for the drug to eliminate the symptoms of a ‘Hypnic’ headache is not exponentially distributed