

<p>JAN 2012</p> <p>The records at a passport office show that, on average, 15 per cent of photographs that accompany applications for passport renewals are unusable.</p> <p>Assume that exactly one photograph accompanies each application.</p> <p>(a) Determine the probability that in a random sample of 40 applications:</p> <p>(i) exactly 6 photographs are unusable;</p> <p>(ii) at most 5 photographs are unusable;</p> <p>(iii) more than 5 but fewer than 10 photographs are unusable. <i>(7 marks)</i></p> <p>(b) Calculate the mean and the standard deviation for the number of photographs that are unusable in a random sample of 32 applications. <i>(3 marks)</i></p> <p>(c) Mr Stickler processes 32 applications each day. His records for the previous 10 days show that the numbers of photographs that he deemed unusable were</p> <p style="text-align: center;">8 6 10 7 9 7 8 9 6 7</p> <p>By calculating the mean and the standard deviation of these values, comment, with reasons, on the suitability of the $B(32, 0.15)$ model for the number of photographs deemed unusable each day by Mr Stickler. <i>(4 marks)</i></p>	<p>JUNE 2013</p> <p>An auction house offers items of jewellery for sale at its public auctions. Each item has a reserve price which is less than the lower price estimate which, in turn, is less than the upper price estimate. The outcome for any item is independent of the outcomes for all other items.</p> <p>The auction house has found, from past records, the following probabilities for the outcomes of items of jewellery offered for sale.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Outcome</th><th>Probability</th></tr> </thead> <tbody> <tr> <td>Item does not achieve its reserve price</td><td>0.15</td></tr> <tr> <td>Item achieves at least its reserve price</td><td>0.85</td></tr> <tr> <td>Item achieves at least its lower price estimate</td><td>0.50</td></tr> <tr> <td>Item achieves at least its upper price estimate</td><td>0.175</td></tr> </tbody> </table> <p>For example, the probability that an item achieves at least its lower price estimate but not its upper price estimate is 0.325.</p> <p>A particular auction includes exactly 40 items of jewellery that may be assumed to be a random sample of such items.</p> <p>(a) Use binomial distributions to find the probability that:</p> <p>(i) at most 10 items do not achieve their reserve prices; <i>(1 mark)</i></p> <p>(ii) 25 or more items achieve at least their lower price estimates; <i>(2 marks)</i></p> <p>(iii) exactly 2 items achieve at least their upper price estimates; <i>(2 marks)</i></p> <p>(iv) more than 10 items but fewer than 15 items achieve at least their reserve prices but not their lower price estimates. <i>(4 marks)</i></p> <p>(b) How many of the 40 items of jewellery would you expect to achieve at least their reserve prices but not their upper price estimates? <i>(2 marks)</i></p>	Outcome	Probability	Item does not achieve its reserve price	0.15	Item achieves at least its reserve price	0.85	Item achieves at least its lower price estimate	0.50	Item achieves at least its upper price estimate	0.175
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<p>JUNE 2015</p> <p>Customers at a supermarket can pay at a checkout either by cash, debit card or credit card.</p> <p>(a) The probability that a customer pays by cash is 0.22 .</p> <p>Calculate the probability that exactly 2 customers from a random sample of 24 customers pay by cash. [3 marks]</p> <p>(b) The probability that a customer pays by debit card is 0.45 .</p> <p>Determine the probability that the number of customers who pay by debit card from a random sample of 40 customers is:</p> <p>(i) fewer than 20;</p> <p>(ii) more than 15;</p> <p>(iii) at least 12 but at most 24. [6 marks]</p> <p>(c) The random variable W denotes the number of customers who pay by credit card from a random sample of 200 customers.</p> <p>Calculate values for the mean and the variance of W. [3 marks]</p>	<p>JUNE 2011</p> <p>An amateur tennis club purchases tennis balls that have been used previously in professional tournaments.</p> <p>The probability that each such ball fails a standard bounce test is 0.15 .</p> <p>The club purchases boxes each containing 10 of these tennis balls. Assume that the 10 balls in any box represent a random sample.</p> <p>(a) Determine the probability that the number of balls in a box which fail the bounce test is:</p> <p>(i) at most 2; <i>(1 mark)</i></p> <p>(ii) at least 2; <i>(2 marks)</i></p> <p>(iii) more than 1 but fewer than 5 . <i>(3 marks)</i></p> <p>(b) Determine the probability that, in 5 boxes, the total number of balls which fail the bounce test is:</p> <p>(i) more than 5; <i>(2 marks)</i></p> <p>(ii) at least 5 but at most 10 . <i>(3 marks)</i></p>										

$P(X = x)$

For exact values we can use the binomial formula or the 'BINOMIAL PD' option on the Casio Classwiz

For each of the following probabilities, mark the students answer.
(They should have given their answer to a minimum of 4s.f.)
Correct any mistakes they have made.

Question	Student Answer	Response
X~B(10, 0.24) find P(X = 3)	0.2429	
X~B(50, 0.09) find P(X = 6)	0.1332	
X~B(18, 0.72) find P(X = 12)	0.3907	
X~B(31, 0.52) find P(X = 16)	0.5532	
X~B(87, 0.35) find P(X = 42)	0.9961	
X~B(60, 0.18) find P(X = 6)	0.0378	
X~B(24, 0.4) find P(X = 10)	0.1611	

NOTATION

$X \sim B(n, p)$

Match each of the correct information to its associated binomial distribution

B(10,0.1)

B(3,0.1)

B(30,0.1)

B(10,0.3)

B(3,0.3)

B(30, 0.3)

n = 3

n = 10

n = 30

p = 0.1

p = 0.3

$P(X < x)$ and $P(X \leq x)$

A the Binomial Distribution is only for **discrete** variables, the upper limit of the probability is vitally important

REMEMBER: $P(X < x) \neq P(X \leq x)$

For Cumulative values we can use the binomial formula, tables or the 'BINOMIAL CD' option of the Casio Classwiz

For each of the following distributions, calculate the probability in the table

$P(X < 11)$	$X \sim B(20, 0.36)$	$P(X \leq 11)$

$P(X < 7)$	$X \sim B(16, 0.19)$	$P(X \leq 7)$

$P(X < 34)$	$X \sim B(56, 0.27)$	$P(X \leq 34)$

CONDITIONS

1.

2.

3.

THE BINOMIAL DISTRIBUTION

Revision Mat

FORMULA

C

X

X

$P(X > x)$ and $P(X \geq x)$

For Cumulative values we can use the binomial formula, tables or the 'BINOMIAL CD' option of the Casio Classwiz

HOWEVER, we must remember that the tables and Casio Calsswiz only calculate cumulative frequencies ($P(X \leq x)$)

REMEMBER: $P(X \geq x) = 1 - P(X < x)$ AND $P(X > x) = 1 - P(X \leq x)$

For each of the following distributions, calculate the probability in the table

$X \sim B(17, 0.49)$	$P(X < 8)$	HENCE: $P(X \geq 8)$

$X \sim B(43, 0.31)$	$P(X \leq 29)$	HENCE: $P(X > 29)$

$X \sim B(32, 0.15)$	$P(X < 12)$	HENCE: $P(X > 12)$

MEAN AND VARIANCE

The mean and variance of a binomial distribution are given in the formula booklet

The **mean** of any Binomial Distribution $B(n, p)$ is _____

The **mean** of a Binomial Distribution is also called the _____

The **variance** of any Binomial Distribution $B(n, p)$ is _____

And hence, the **standard deviation** of any Binomial Distribution $B(n, p)$ is _____

$P(a < x < b)$

For double inequalities you **MUST** ensure you know the values you are being asked to include

For each of the following double inequalities, state the values of X that you are looking to include in the inequality

$P(12 < X < 15)$	
$P(9 < X < 13)$	
$P(24 \leq X < 28)$	
$P(37 \leq X < 42)$	
$P(16 < X \leq 21)$	
$P(3 < X \leq 7)$	
$P(40 \leq X \leq 46)$	
$P(28 \leq X \leq 34)$	